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AN ECONOMETRIC MODEL OF LIMIT PRICING: THE COMPUTER INDUSTRY

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An Econometric Model of Limit Pricing: The Computer Industry

A Dissertation submitted in partial satisfaction of the requirements for the degree of

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in

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by

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To my parents

I would like to acknowledge
the help received from my committee
while working on this dissertation.

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ABSTRACT

An Econometric Model of Limit Pricing:
The Computer Industry

by

John Erwyn Leonard

The problem is to estimate key economic parameters in the computer industry, cast the analysis in a limit pricing mold, and demonstrate the robustness of the model.

Data sets were gathered from the computer publications and general economic periodicals. Economic variables were defined and obtained from observations by appropriate techniques such as first differencing. Econometric analysis of the data validated the limit pricing model and rejected other models.

Using the data and coefficients, the researcher cast the model into a dynamic optimization framework. This was studied by dynamic programming and simulation techniques. Stability and sensitivity analysis were undertaken to verify robustness of the model.

The findings validated the choice of the limit pricing model and the decision to apply it to a short run problem. Econometric estimations of demand and rival market penetration was successful. Both economic and econometric criteria wre met. The dynamic programming algorithms were used to show stability aspects of the dynamic

profile of prices, quantities and profits. Simulation routines confirmed the dynamic programming analysis under uncertainty. To close the simulation, myopic profit maximizing models were run and evaluated. Customary evaluation techniques were employed to demonstrate the superiority of the optimal profile over myopic or actual profiles. Implications for risk averse firms can be drawn. The greater stability of the optimal series is a desirable property in a risk averse firm. This leads to an increase in inventory for such firms. Conclusions of previous researchers were confirmed and their analysis extended to questions of sensitivity and stability.

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INTRODUCTION

References to limit pricing hearken back to a seminal article by Bain [1]. Bain sought to explain the prevalence of prices below the profit maximizing level. This occurred notably in the steel and cigarette industries. Among other possible explanations, he suggests that "established sellers persistently, or 'in the long run,' forego prices high enough to maximize the industry profit for fear of thereby attracting new entry to the industry and thus reducing the demands for their outputs and their own profits."[1] Entry can be looked at from the viewpoint of the established firm(s) or from the viewpoint of the potential entrant (rival), he observes. This latter rival entry response effect can be empirically tested. To treat this formally, Bain introduces the concept of "limit price." In his words this is the "highest common price which the established seller(s) believe they can charge without inducing at least one increment to entry."[2] This can be estimated by the established firm according to its perceptions of potential entrants. Specifically their demands, costs, and behaviors must be estimated. Bain

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[1]Bain (1949), p. 449.

[2]Ibid. p. 454.

recognizes the stochastic element inherent in estimations of other firms' costs, demands, and behaviors. He further suggests several interesting extensions of his analysis. For example, "firms established within an oligopoly may hold price down for fear of 'fattening' their smaller rivals sufficiently to encourage their expansion."[3] Dynamic analysis is proposed, but thought difficult due to many variables which change over time. Refinement of the analysis and its application to various industries was left to later researchers.

Gaskins [2] sought to extend the analysis by employing a continuous framework. Specifically he rejected the dichotomy of limit price or profit maximization price in favor of an optimal price. This optimal price balances current profits and future market shares. If entry depends on price, the dominant firm can control entry as well as its own profits by controlling price. Since entry is a dynamic phenomenon, the use of dynamic optimization is called for. Gaskins postulates the dominant firm maximizes the present value of its own profits subject to entry. The decomposition of industry demand into dominant demand and rival (i.e. competitive fringe) demand makes the interdependence of firms explicit. Limit price

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[3] Ibid. p. 463.

is the price at which net entry equals zero. Pricing below limit price causes exit. Cost advantage is the difference between limit price and the major firm's average total cost. Gaskins makes the assumption that the cost advantage is the difference between limit price and the major firm's average total cost. Gaskins makes the assumption that the cost advantage is positive or zero at worst. He shows that the optimal price in his framework is below the myopic profit maximizing price. His comparative static analysis shows that limit price is inversely related to equilibrium rival entry and to optimal price. As cost advantage increases, the dominant firm has more incentive to drive out rivals and later raise price (if the limit price increases).

The presence of market growth reinforces the model. Here the equilibrium optimal price exceeds the limit price. A firm with no cost advantage can retain market share. Gaskins assumes a dynamic demand curve and finds optimal price and market share for various levels of growth. Even without cost advantage, high market share can be maintained in a high growth environment. In the growth case, an increase in limit price increases equilibrium price for static analysis. For comparative dynamics, the effect is reversed. The policy analysis using limit pricing models lead to conflicting results. Encouraging entry by patent licensing would lower limit price, long run price, and market share, but increase short run price. However, encouraging entry by lessening the information costs or capital market disadvantages would lower short and long run prices while raising market share of the dominant firm if it has sizable cost advantages. With this mathematical and policy analysis, only the theoretical model is given. An empirical application is lacking.

Rifkin and Sengupta [3] further update and modify limit pricing analysis. An innovation in their formulation is the use of a stochastic term in the inverse demand function. Quantity is used as the control variable. Simulation techniques are used to provide a dynamic profile of leading industry variables. Risk aversion is assumed in the formulation. The limit price is assumed constant over time. Behavioral aspects of the response coefficient are discussed. This response coefficient details how rivals respond to an excess of market price over limit price. The simulations presented assume limit price greater than cost (Gaskin's positive cost advantage case). The authors discount both future profits and future entry, whereas Gaskins discounted only future profits. Growth in demand is assumed and varied in the simulations. Other parameters varied include discount rate, demand slope, and response coefficient. Stability analysis demonstrates the robustness of the model. Paralleling Gaskins, optimal output is shown to exceed myopic profit maximizing output. To deter entry the dominant firm could increase output capacity via investment. In summary, Rifkin and Sengupta's switch to quantity determines an optimal output policy for maximum discounted expected value. This limit price theory explains the persistence of excessive profits, but not the precise long run market share. Limit pricing is a behavioral guide, not an exact prescription.

The application of limit pricing to certain industries is obvious. Harris [4] affirms that limit pricing can model entry well. He found a close fit for metals and machinery. In 1969 the U.S. Department of Justice filed suit against IBM charging it with "introducing 'selected computers, with unusually low profit expectations, in those segments of the market where competitors had unusual competitive success' in order to restrain competitors from entering or remaining in the general purpose computer market." [5] This indicates limit pricing and the government's economic witness, Alan McAdams, testified that IBM engaged in "death level pricing."[4] Other sources exonerate IBM and even imply that limit pricing is an aid to competitors [6]. Brock [7] built an optimal model for the

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[4]Reported in an untitled article by C. Arnst, <u>COMPUTERWORLD</u>, 9/19/77, p. 14.

computer industry based on Gaskins' formulation. Since the industry is marked by technical progress, Brock computes a technical progress rate and employs this in the model. The limit price is assumed to decline exponentially at this rate, as does the average total cost of production. Further, the response coefficient increases at this exponential rate. The model implies this rate is negatively related to market share in the long run. In addition, demand growth stimulates the firm to retain market share at the expense of myopic profit considerations. Finally, in contrast to Gaskins simulations, the major (or dominant) firm will lose market share without cost advantage even if market demand grows rapidly. Brock assumes the IBM price is close to the limit price because IBM has not lost market share quickly. Averaging IBM's excess profit margin and cost advantage (from product differentiation and economies of scale) he estimates the difference between IBM's price (hence the limit price) and costs at 15%. This particular limit pricing model does not disaggregate to various firms or address matters of computer generations. Matters external to the Gaskins-Brock model dictate a maintenance of market share -- the high discount rate suggests no decrease, and difficulty in switching companies mitigates against any increase in market share.

The Brock model can be reformulated in several ways.

More data are now available and different approaches can be used, some of which also employ limit pricing. Although the data set is still imperfect, it can support a quarterly econometric study. Key parameters can be more closely estimated on a quarterly basis than in Brock's annual estimation. For example, the response coefficient of Gaskins can be properly estimated in econometric equations. This is made more comprehensible by the disaggregated data now available.

A competitive fringe can be replaced by one or more bona fide rivals.

As per Bain (op. cit.) the matter of defense of market share can be studied. This involves a definition of market penetration for rivals. As this market penetration variable (increase of production) is a limited proxy, it is used for estimation in the short run. Unfortunately, more directly observable entry is not found in any reported data. Any such attempt at entry measurement would be frustrated by lack of observations and poor quality of those noticed. Further, technology change and its effect on the industry cast doubt on the value of long run work. The Brock solution involves arbitrary data selection and is only acceptable as a stop gap measure.

By contrast the quarterly model can be analyzed for sensitivity and stability. Thus it can be studied for robustness across data sets and values of important parameters.

Dynamic programming can illustrate the sensitivity to variations in parameters or even basic behavioral assumptions. It can further show the effect upon optimal trajectories when these assumptions or parameters are changed.

Simulation compares actual and optimal paths for this reformulation of the Gaskins-Brock model. Certain simulation routines can also be compared in the long run and short run paths. The economic rules of the limit pricing scenario can be checked carefully. Such matters as optimality and stability can be applied to this limit price formulation of the computer market. While the actual data exhibits wide variation, an optimal path would be smoother and call for smaller inventory expense. Risk aversion is consistent with this observation. Other sub-optimal policies can be evaluated with these tools (simulation and dynamic programming). Comparison with other models will show their applicability to a short run computer study.

In summary, general analysis (as in simulation) and partial analysis (via econometric estimation) will be used to construct and study a short run limit pricing model of the computer industry.

Other models in the literature can also be considered for the computer industry. Its structure of a dominant firm with smaller rivals suggests a variety of applicable models. A dominant firm price leadership analysis was considered but found unacceptable on economic grounds. Similarly the classic supply and demand paradigm was rejected. Analysis based on competitive superiority has been proposed. Inasmuch as no distinguishing behavior sets this latter model apart, it can only be mentioned parenthetically. The available data observations are cast into these frameworks for analysis. The ensuing econometric estimation implies the difficulties in retaining these other models. The reaction curves for the price leadership model exhibit explosive tendencies (rivals increase production faster when the dominant firm's production increases). Supply and demand analysis yields such phenomena as downward sloping supply curves. Accordingly the analysis has been narrowed to the limit pricing model.

The objectives of the dissertation may be summarized briefly:

- First, we econometrically estimate key structural coefficients for the computer industry.
- Second, these econometric coefficients are applied

to a short-run limit-price model.

- Third, the model is studied for economic viability when key parameters are varied.

- Finally, the model is compared to one featuring myopic profit maximization.

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6. Spiegel, J. "Telex vs. IBM: Monopoly Pricing under Section 2 of the Sherman Act" The Yale Law Journal Vol 84, January 1975, pp. 558-583.

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SURVEY OF THE LITERATURE

Bain [1] studied twenty industries from the view of difficulty of entry. He classified the industries according to the relation of profit maximizing price, average cost, and entry deterring (limit) price. He then sought to explain the difficulties of entry in those industries in which entry deterring price exceeded average cost. The economic explanation involves product differentiation and economies of scale which, in turn, have other economic explanations (brand loyalty, marketing technology, etc.).

Modigliani [2] not only summarizes the Bain work, but also provides an instructive graphical apparatus for analysis of demand and entry in such markets. His contribution further includes algebraic analysis of the curves and specific mention of the behavioral assumptions implicit in such models such as warfare or price maintenance. The analysis uses market demand and established firm's cost to locate critical prices and quantities which would deter entry. Modigliani has thus made a major contribution in his exposition and extension of Bain's work.

For the theoretical underpinnings it is also necessary to understand the mathematical versions of limit pricing. One of the seminal articles is that of Gaskins [3].

Since Gaskins' work is so often cited and so central to this development, it will be presented in detail. Gaskins casts limit price theory in a dynamic, continuous time setting. His equations are then amenable to solution via optimizing techniques and differential equations methods. Comparative statics and dynamics techniques then delineate signs of differentials, thus permitting policy statements. Gaskins[1] first states profit in equation (1):

$$V=\int_0^\infty [p(t)-c]q(p(t),t)e^{-rt}dt, \tag{1}$$

where V is the sum of discounted profits p(t) the price at time t, c the average total cost, q the dominant firm's quantity, and r the discount rate. Next q is expressed as

$$q(p(t),t)=f(p(t))-x(t). \tag{2}$$

Entry is then quantified as

$$\dot{x}=k[p(t)-\bar{p}] \tag{3}$$

i.e. change in rival firms' output is proportional to excess of market price over the limit price $\bar{p}$. The constant of proportionality k is the response coefficient. It is assumed that x comes from a competitive fringe. Substituting (2) into (1) gives equation (4)

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[1]Gaskins (1971), p. 307 ff.

$$V=\int_0^{\infty}[(p(t)-c)(f(p)-x(t))]e^{-rt}dt \tag{4}$$

to be maximized subject to the entry equation (3). This can be done via Pontryagon techniques. A Hamiltonian is set up and necessary and sufficient conditions for an optimum are obtained. Thus Gaskins sets

$$H=(p(t)-c)(f(p)-x(t))e^{-rt}+Z(t)k(p(t)-\bar{p}) \tag{5}$$

for which it is necessary for the optimal (maximal) solution that there exist an adjoint variable Z such that

$$\dot{x}^*(t)=k(p^*(t)-\bar{p}),\ x^*(0)=x_o \tag{6}$$

$$\dot{Z}^*(t)=-\frac{\partial H}{\partial x}\ ,\ \lim_{t\to\infty} Z^*(t)=0 \tag{7}$$

$$\frac{\partial H}{\partial p}=0. \tag{8}$$

Sufficiency for the maximal optimal solution is assumed by requiring profit to be a smooth concave function of price along f(p) the industry demand function. This insures H is concave with respect to p and x. From (8) Gaskins finds

$$Z^*(t)=\frac{(x(t)-f(p)-(p^*(t)-c)f'(p))e^{-rt}}{k} \tag{9}$$

and using this along with (6) and (7) finds by use of differential equations

$$\dot{p}= \frac{k(p-c)+r[x-f(p)-(p-c)f'(p)]}{-2f'(p)-(p-c)f'(p)} \tag{9a}$$

$$\dot{x}=k(p(t)-\bar{p}),\ x(0)=x_o. \tag{6}$$

Since the transversality condition provided Gaskins no information, he argued existence and uniqueness of the optimal trajectory by phase diagrams in x-p space. He proves the (myopic) profit maximizing price is non-optimal. A solution for equilibrium values of x and major firm market share s is obtained. Specifically they are solved for as functions of limit price $\bar{p}$, average total cost c, discount rate r, and response coefficient k. Next Gaskins differentiates to obtain comparative statics results. Off the optimal path he sets $G = \frac{dp}{dx} = \dot{p}/\dot{x}$ and demonstrates signs for partials from $p^* = f(r, c, k, \bar{p})$. To make demand as well as supply (via entry) dynamic, a demand curve reflecting market growth is postulated. Here

$$q(p(t),t)=f(p(t))e^{\gamma t}-x(t) \tag{10}$$

and

$$k=k(t)=k_o e^{\gamma t} \tag{11}$$

thus the response coefficient also is made dynamic. A transformation $w(t)=x(t)e^{-\gamma t}$ scales this new system so that the previous analysis is applicable. Unfortunately sufficiency can no longer be shown. The hypothetical model

$$q=(100-p(t))e^{\gamma t}-x(t)$$

with $c=10$, $r=.1$, $k_o=1$ and $x_o=0$ is shown for $\bar{p}=15$ (cost advantage), $\bar{p}=10$, and growth rates up to 8%. Tabulated are equilibrium prices and market shares (of the dominant firm). Gaskins concludes with the model's policy implications.

A model revising and building upon the Gaskins analysis is that of Rifkin and Sengupta [4]. The authors employ quantity as their control variable. The demand curve is introduced as dynamic, thus necessitating a transformation of variables for analysis. Stability analysis is used and convergent paths are simulated. The dynamic price term is

$$p(t)=ae^{nt}-b_1\ (q(t)+x(t))+u \qquad (1\ \text{R\&S})$$

where a is the initial intercept, n the positive proportional rate of market growth, b_1 the demand slope, q the dominant firm's quantity, x the rival firms' quantity, and us the error term. Entry is similar to Gaskins' form

$$\dot{x}=k(p(t)-\bar{p})=k[ae^{nt}-b_1\ q(t)-b\ \ x(t)-\bar{p}]. \qquad (2\ \text{R\&S})$$

To allow for the stochastic price term expected profits are computed with $E(\pi(t))=[E(p(t))-c]q(t)$ and variance

$\pi(t)=Vq^2(t)$ where c is the average total cost, V is the variance, and E the expectations operator. Profits may be discounted and summed as

$$V=\int_0^\infty e^{-rt}[(ae^{nt}-b_1q(t)-b_1x(t)-c)q(t)-mVq^2(t)]dt \quad (3\ R\&S)$$

where m is a positive constant reflecting risk aversion. Combining terms $b_2 = V+b_1$ and

$$V=\int_0^\infty e^{-rt}[ae^{nt}-b_1x(t)-b_2q(t)-c]q(t)dt \quad (4\ R\&S)$$

To optimize they use the Hamiltonian

$$He^{-rt}=e^{-rt}[(ae^{nt}-b_1x(t)-b_2q(t)-c)q(t)+kZ(t)$$
$$(ae^{nt}-b_1x(t)-b_1q(t)-\bar{p})] \quad (5\ R\&S)$$

in which Z is the adjoint variable. Notice both profit and entry are discounted. The necessary conditions are that there exists a Z such that

$$\dot{x}^*(t)=k(ae^{nt}-b_1x(t)-b_1q(t)-\bar{p}) \quad (6\ R\&S)$$

$$\dot{Z}^*(t)=-\frac{\partial H}{\partial x}+rZ(t); \quad \lim_{t\to\infty} Z^*(t)=0 \quad (7\ R\&S)$$

$$\frac{\partial H}{\partial q}=0. \quad (8\ R\&S)$$

Observe (7 R&S) can be rewritten as

$$\dot{Z}^*(t)=b_1q(t)+(b_1k+r)Z^*(t) \quad (9\ R\&S)$$

Transforming to scale down values from infinite growth has

$$x(t) = \tilde{x}(t)e^{nt}, \quad \tilde{q}(t) = qe^{nt}, \quad z(t) = \tilde{z}e^{nt}.$$

The necessary conditions thereby become

$$\dot{\tilde{x}}^*(t) = -(kb_1+n-\frac{kb_1{}^2}{2b_2})\tilde{x}^*(t) + \frac{k^2b_1{}^2}{2b_2}\tilde{z}^*(t)+A_1 \qquad (10\ R\&S)$$

$$\dot{\tilde{z}}^*(t) = \frac{-b_1{}^2}{2b_2}\tilde{x}^*(t)+(kb_1+r-n-\frac{kb_1{}^2}{2b_2})\tilde{z}^*(t)+A_2 \qquad (11\ R\&S)$$

where $A_2 = \frac{ab_1-cb_1e^{-nt}}{2b_2}$ and $A_1 = ka-\frac{kab_1}{2b_2}-e^{-nt}(k\bar{p}+\frac{kb_1c}{2b_2})$.

Rifkin and Sengupta then proceed with stability analysis. The characteristic equation

$$f(\lambda) = \lambda^2+(2n-r) +(n^2-nr-b_1kr-b_1k^2+\frac{rb_1{}^2}{2b_2}k+\frac{b_1{}^3k^2}{b_2}) = 0 \qquad (12\ R\&S)$$

has roots λ_1 and λ_2 expressed as

$$\lambda_1 = -(n-r/_2)+\tfrac{1}{2}\sqrt{4(b_1kr+b_1{}^2-k^2-\frac{k^2b_1{}^2r}{2b_2}-\frac{k^2b_1{}^3}{2b_2})+r^2} \qquad (13\ R\&S)$$

$$\lambda_2 = -(n-r/_2)-\tfrac{1}{2}\sqrt{4(b_1kr+b_1{}^2k^2-\frac{k^2b_1{}^2r}{2b_2}-\frac{k^2b_1{}^3}{b_2})+r^2} \qquad (14\ R\&S)$$

If m = 0, this lack of risk aversion means $b_2 = b_1$ which reduces λ_1 and λ_2 to

$$\lambda_1 = -(n-r/_2)+\tfrac{1}{2}\sqrt{r^2+2bkr} \qquad (15\ R\&S)$$

$$\lambda_2 = -(n-r/_2)-\tfrac{1}{2}\sqrt{r^2+2bkr} \qquad (16\ R\&S)$$

Further, if in addition $r = 0$, then $\lambda_1 = \lambda_2 = -n$. This no discount case implies the system must decay, else $\lambda_1 = \lambda_2 = 0$.

The entire solution for $\tilde{x}^*(t)$ and $\tilde{z}^*(t)$ may be written as follows

$$\tilde{x}^*(t) = B_1 e^{\lambda t} + B_2 e^{\lambda_2 t} + e^{-nt}\frac{(bkc+rc-bk\bar{p})}{rb} + \frac{ak(r-n)}{bkr+2nr-2n^2} \qquad (17\ R\&S)$$

$$\tilde{z}^*(t) = \frac{(bk+r+(r^2+2bkr)^{\frac{1}{2}})}{bk_2} B_1 e^{\lambda_1 t} + \frac{(bk+r-(r^2+2bkr)^{\frac{1}{2}})}{bk^2} B_2 e^{\lambda^2 t}$$

$$+ \frac{(c-\bar{p})}{r} e^{-nt} - \frac{an}{bkr+nr-2n^2} \qquad (18\ R\&S)$$

The authors discuss convergence and stability in the light of these results. Rifkin ran simulated trajectories which they tabulate next. All runs are required to be stable a priori. As Gaskins showed myopic profit maximizing price to be non-optimal, Rifkin and Sengupta also show myopic profit maximizing quantity to be too small. The effect of risk aversion is considered with its possibility of leading to a positive characteristic root thus (via transversality) causing a reduced system. An alternative decomposition of the system is considered using $\tilde{x}(t)$ and $\tilde{q}(t)$

$$\dot{\tilde{x}}^*(t) = -(kb_1+n)\tilde{x}^*(t) - kb\,\tilde{q}(t) + ka - \bar{p}e^{-nt} \qquad (19\ R\&S)$$

$$\dot{\tilde{q}}^*(t) = \left(\frac{2kb_1{}^2+rb_1}{2b_2}\right)\tilde{x}^*(t)+(b_1k+r-n)\tilde{q}^*(t)+A \qquad (20\ R\&S)$$

where $A = \frac{n-r-b_1k(a-ce^{-nt})}{2b_2}+\frac{(kb_1\bar{p}e^{-nt}-kab_1)}{2b_2}$

These can be seen as dynamic reaction curves with cross variation terms

$$\frac{\partial\dot{\tilde{x}}^*(t)}{\partial\tilde{q}^*(t)} = -kb_1 \text{ and } \frac{\partial\dot{\tilde{q}}^*(t)}{\partial\tilde{x}^*(t)} = \frac{2kb_1{}^2+rb_1}{2b_2}$$

Comparative statics results are obtained from steady state price and market share equations. These are then compared with standard literature results as in Gaskins. They note the exponential market growth response formulation of Gaskins which they reject. They also suggest that limit price theory does not predict terminal market share, in contrast to other work.

Chow [5] specifically studies computers in an economic context. Since cost data is fragmentary at best, he concentrated on the dynamic demand for computers. His work concentrates on technological change and growth in the computer industry. His theory involves the use of differential equations to analyze growth and demand over time. (Chow corrects for quality change by using hedonic price indices.[2] Inflation is corrected by the use of the

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[2]See below, chapter 3, pp. 57-61 for specifics.

GNP deflator. Industry data is not broken down to the firm level.) The model is a stock-adjustment model in the sense that Chow assumes the rate of growth depends on a stock effect and on an equilibrium adjustment effect. Demand then depends upon price, quantity lagged, and income. Quantity is taken in value form: thousands of dollars paid as monthly rental fees. Then smoothed price is used to construct a price index. The specific equations will show the entire process in detail. Chow begins with a differential equation representing the growth process.

$$\frac{dy}{dt} = \alpha y(\log y^{*} - \log y) \qquad \text{(C-1)}$$

or

$$\frac{d\log y}{dt} = \alpha(\log y^{*} - \log y) \qquad \text{(C-2)}$$

He approximates the derivative of log Y by difference techniques and the extant stock by U_{t-1}. Thus the equation assumes the difference equation form

$$\log Y_t - \log Y_{t-1} = \alpha(\log Y^{*} - \log Y_{t-1}) \qquad \text{(C-3)}$$

From static demand theory he asserts

$$\log Y_t^{*} = B_o - B_1 \log P_t + B_2 \log X_t \quad \text{output} \qquad \text{(C-4)}$$

which can be substituted into the above

$$\log Y_t - \log Y_{t-1} = \alpha B_o - aB_1 \log P_t + \alpha B_2 \log X_t - \alpha \log Y_{t-1} \qquad \text{(C-5)}$$

Rearranging by logarithmic procedures gives

$$\log {}^{Yt}/_{Yt-1} = C - C_1 \log P_t - C_2 \log Y_{t-1} + C_3 \log X_t + U_t \qquad \text{(C-6)}$$

with U_t a theoretical error process and the coefficients given their expected signs. The proxy for X_t (products of firms using computer services as inputs) was GNP and it proved unavailing. Accordingly it was omitted in the final equation used for validation. The accepted estimation was

$$\log {}^{Y}t/_{Y_{t-1}} = 2.950 - \underset{(.1726)}{.3637} \log P_t - \underset{(.0739)}{.2526} \log Y_{t-1}$$

The quotient of the coefficients on log P_t and log Y_{t-1} is 1.44 which in this log-linear estimation can be used to estimate the price elasticity of demand. The coefficients on log P_t and log Y_{t-1} are statistically significant at the level of 5%. (In comparing the actual series in log ${}^{Y}t/_{Y_{t-1}}$ with the theoretical one estimated above, he finds a sizeable error in the year 1961. This is a pivotal year in the transition from first generation (vacuum tube) to second generation (transistor) computers. The adjacent years (1960 and 1962) also show large errors which Chow attributes to delivery delay.) It is noteworthy that this is an annual study covering twelve years. The construction of the hedonic price indices required pooling of data. In spite of the problems, Chow constructed useful series and performed interesting estimators.

Sharpe [6] mentions the work of Chow and others. His book first summarizes much microeconomic theory, then applies it to the world of computers. A discussion of the industry breaks it down into various parts (firms, brokers, etc.). The history is presented in copious detail for the product and the industry. Data sources and product characteristics are discussed. The matter of rental versus sale is presented along with the relevant economic and legal issues. Computers are analyzed in terms of cost to consumer and effectiveness in accomplishing tasks. While price is given (i.e. cost to user), cost of making computers by firms is not thoroughly treated. Questions of memory and computer services (as in time sharing) are studied. A concluding chapter on the computer market with attendant services and costs ends Sharpe's work. Material ranges from economic analysis to computer science, from technical progress to Grosch's law.[3]

Brock [7] writing in 1975 updates the Sharpe

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[3]Sharpe (1969) p. 315.

analysis with further developments in legal and technical areas. Also he takes the Gaskins theory and applies it to the computer world of Chow. He extends the Gaskins analysis by adding terms representing technical progress. Also his basic framework is the log-linear function. Thus average total cost and limit price decline exponentially over time. As with Gaskins, profit, but not entry, is discounted. The Brock analysis also relies heavily on data from Chow. He explains Chow's use of hedonic price indices and uses Chow's series for price and quantity. Technical progress is estimated by relating price (which declines, presumably reflecting technological change) to the explanatory variable time. The resulting coefficient is used as the exponential coefficient of decline for the limit price and average total cost terms. By studying economies of scale and excess profit margins Brock obtains a 15% cost advantage for the dominant firm in the computer industry, IBM. Thus cost is set equal to 85% of the limit price. Notwithstanding occasional declines (and an overall decline) in IBM market share, it is assumed that IBM is limit pricing during the study.[4] The response coefficient (recall Gaskins above) is obtained by choosing two years of market share data for IBM in which the earlier figure

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[4]The alternative explanation of competitive superiority can not be ruled out.

exceeds the latter. This decline is placed in an elementary equation to obtain the value for k. An equation from Gaskins was estimated and used to get a discount rate. Validation proceeds by comparing actual data (Chow indices, Honeywell vs. Rand market share data) with that estimated by his equations. No disaggregation to firms is attempted nor is the matter of computer generations analyzed. The response rate is assumed to rise exponentially at the rate of technical progress (cf Gaskins, Rifkin and Sengupta). As this is a response rate for entry, the rise in IBM market share which occurs from time to time suggests a problem with any simple interpretation. Perhaps entry and exit phases of the analysis could be studied. The demand (recall, a rental value) likewise proceeded with difficulty. A high correlation of price and time caused a modification of the equation. Thus modified quantity value was dependent on time. Brock used Chow's price elasticity of demand to aid the estimation:

$$\log S + 1.44 \log P = 9.225 + .147T \qquad (B\text{-}1)$$
$$(6.08)^{**}$$

is estimated with an R^2 of .82 and the t-value is significant at the one percent level. This misses the stock and (unsuccessful) income terms in Chow's estimation. Brock next estimates the rate of technical progress:

$$\log P = 1.54 - .237T \quad R^2 = .99$$
$$(27.2)^{**}$$

yielding the value .237 for use in his exponential equations. Observe the earlier value is a growth rate (.147T). A major disadvantage of the Brock work is the entry estimation. The theoretical equation (c.f. Gaskins, Rifkin and Sengupta) is

$$\dot{x} = k_0 e^{at}(p_t - \bar{p}) \tag{B-2}$$

Since a is the technical progress term estimated above, p_t is Chow's price index, $\bar{p} = \bar{p}_o e^{at}$ and $\bar{p}_o$ is set $= p_o$, the critical estimation is that of k_o. Since Brock does not disaggregate the industry, the aforementioned method of subtracting market shares is used to gain (a proxy for) entry. The years chosen for market scrutiny are 1963 and 1965. The latter is chosen (correctly) as a low point in the series for IBM market share. The former Brock calls the high point of IBM's market share. Barring numerous misprints in the series it is not a maximum in the series or even in the subseries for the second generation. Accordingly that choice is somewhat arbitrary, thus his estimation is similarly flawed. A presentation in detail will highlight the problems. His table gives 1963 share = 69.8% and 1965 share of 65.3%, the difference being 4.5% as a measure of IBM loss or rival firm entry. Since these are the seventh and ninth years in the series, Brock sets

$$\int_7^9 \dot{x}(t)dt = .045. \qquad \text{(B-3)}$$

Recall his equation for entry and set $p(7) = \bar{p}(7)$ which yields the integral equation

$$.045 = \int_7^9 k_o e^{.237t}(2.53e^{-.237(7)}-2.53e^{-.237t})dt \qquad \text{(B-4)}$$

where a = .237 as estimated above and 2.53 = Chow's price index level for 1956. The solution proceeds as follows:

$$.045 = \int_7^9 K_o e^{.237(t-)}2.53dt-\int_7^9 k_o e^{O}2.53dt \qquad \text{(B-5)}$$

$$.045+\int_7^9 2.53K_o dt = 2.53K_o\int_7^9 e^{.237(t-7)}dt \qquad \text{(B-6)}$$

$$.045+5.06K_o = 2.53K_o\int_7^9 e^{.237(t-7)}dt \qquad \text{(B-7)}$$

$$.045+5.06K_o = \frac{2.53K_o}{.237} e^{.237(t-7)}\Big|_{t=7}^{t=7} \qquad \text{(B-8)}$$

$$.045+5.06K_o = \frac{2.53K_o}{.237} (e^{.474}-1) \qquad \text{(B-9)}$$

$$.045+5.06K_o = \frac{2.53K_o}{.237} (.6064) \qquad \text{(B-10)}$$

$$.045 = 1.4135K_o \qquad \text{(B-11)}$$

$$.03 = K_o \qquad \text{(B-12)}$$

This solution was presented in laborious detail; Brock omits the equations (b-5) to (B-11). However a true maximum maximorum is in 1957 in which IBM's market share was 78.5%. Substituting this into the equation the difference is .132 thus

$$.132 = \int_2^9 \dot{x}(t)dt \qquad \text{(B-13)}$$

yields as above K_o = .005. On the other hand let the years chosen be 1965 and 1967. The rise in IBM's market share as the third computer generation (based on integrated circuits) began is from 65.3% to 68.1%. The difference $-.028 = \int_7^{11} x(t)dt$ yields k_o = -.02 for an exit coefficient. Observe that a rise in IBM's market share means a decline in rival market penetration here. The highest entry coefficient calculated in this manner is .04 from 1958 to 1960. The range of calculated K_o's declines to the earlier mentioned exit coefficient of .02. Accordingly any such estimation of k_o is suspect without further background justification. Product generation history would limit this analysis, the more so as the suggested pattern (IBM gains market share as a generation starts, loses it at its end) is not tenable in all generations. While this theory has adherents, it also has opponents and the data set doesn't support it.

Finally, there is the estimate of the discount rate. Using estimates for the growth coefficient and other parameters

$$r = .25128 + \frac{.0144006}{s}. \qquad \text{(B-14)}$$

Since IBM can have no more than 100% of the market, r has a lower bound of .26568. For a market share = 65.4%, r = .2733, the value that Brock accepts. In graphical comparisons of the actual data with values estimated from his equations, the Brock work comes out fairly well. Further, his predictions are borne out fairly well by subsequent developments in the industry.

Updating can be done from any of several sources. While some data series have been discontinued, other materials exist and even update material for comparison with Chow's and Brock's work. The International Data Corporation report and forecast for the industry [8] gives short data series, product and technology discussions, economic predictions, and forecasts until 1983. While the extrapolation of Brock's data via this and other sources is possible, it would be difficult. The industry discussion here is more valuable to stockbrokers and computer engineering specialists. Phister [9] provides an encyclopedic discussion of the industry with much useful data. Market share is carried forward to 1974 as is value of orders.

Using supplementary data available on production, unfilled orders, and inventory, many interesting annual studies could be seen for the period 1956-1974; the data series generally cease in 1974.

Finally, there is the question of model evaluation once such data have been employed to construct parametric equations to estimate key industry values. Simulations can be done as in [2] above. They can give an excellent insight into speed of convergence. Further phases for the strategy space were marked out. Also the response when the strengths of various effects are changed can be measured. This is confirmed in [1] above. There are several methods available for doing simulations. Successful simulation work has been done by physical experimentation; alternatively computers are used for more complex work.[5] Naylor et al. [10] have considered methods of evaluating simulations and other estimations. When controlled or ex post experimentation is impossible, econometric is used for a simple system; numerical analysis or simulation is required for nonlinear stochastic

[5] Some programs for simulation are written in FORTRAN and are eminently usable. Canned programs can also be used for simulation work. DYNAMO is a continuous time stochastic simulation language. It is easy to use and provides quick insights for simple problems. A powerful discrete time stochastic simulation program is SIMSCRIPT II.5 While the SIMSCRIPT package is more complex and costly to run, it does provide superb formatting and powerful capa-

systems. Simple calculation of means is inadequate for evaluating the success of simulations. Naylor et al. list numerous methods of evaluation. Goodness of fit can be studied by a variety of measures including correlation coefficients. [11] Suffice it to say, a variety of methods of evaluation is available to the economist seeking to validate a model.

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bilities. Even such a simulation must still be evaluated, usually with respect to an actual series. Thus a simulated series is compared to an "actual" series.

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THE EMPIRICAL MODEL: SPECIFICATIONS AND DATA CHARACTERISTICS

Economic relationships in the computer industry have generated considerable discussion and little agreement. A commanding market share by IBM is evidence of monopoly or market power to some observers.[1] Others find the shock effect of actions by small firms such as Itel and Amdahl indicative of the competitive nature of the industry. IBM is at best, first among equals in the small business computer segment and is one of the small fry in mini-computers. Indeed DEC holds a predominant position among the minicomputer manufacturers. Over the course of time the market share of IBM in the GPC market has been rather stable since the industry was considered viable in 1956. This seems a little surprising since Remington Rand had the early (1950) lead in the industry. A pattern of IBM and its competitors was likened to "Snow White and the Seven Dwarves." Although IBM had other competitors besides the seven (Burroughs, Control Data, Honeywell, NCR, Sperry Rand, RCA - since exited, and GE-since exited), the seven were major

[1]Other economists suggest that competitive superiority can explain IBM's behavior.

companies in computers with competitive staying power. In spite of their size, RCA and GE were forced to withdraw from the market due to marketing problems, innovations, and the like. Smaller firms have sought various niches in the industry and have been aided by the concept of plug compatible equipment. Such firms are known as PCM's (Plug compatible manufacturers). The PCM's together are roughly the size of a "dwarf," but they have had considerable impact on the industry recently. When Amdahl and Itel announced new machines with immediate delivery, their stock soared. IBM made a statement predicting performance and prices on its proposed series and the PCM's were devastated. Differences in software, service contracts, and data maintenance imply significant costs of changing manufacturers. The PCM's have only partially overcome this with their plug compatibility. Some analysists have sought to further subdivide the market, believing that competition occurs in smaller sub-markets. Thus the GPC sector could be subdivided into supercomputers (Control Data, Cray) general service computers (IBM, Honeywell, Sperry Rand) and specialty machines (NCR, Burroughs). This approach is not widely accepted at present and will not be employed. The matter of action and reaction on an intra-industry basis is a matter of much speculation. As per the introduction, not even the conduct of IBM is agreed on by economists studying the industry. Structure, then, remains stable; behavior, conduct, and performance are more subjective.

Economic relations are further obscured by technical progress. This merely accentuates the heterogeneity of the product called "computer." At least five generations of computers can be traced in the last forty years. Also at any given time several generations may be viable as installed equipment. The product is further differentiated by accessability of software, levels of service support, etc. Measurement of various characteristics can be used to define a uniform product by correcting for such diversity of hardware. This is done by using hedonic price index construction.[2] Thereby demand for characteristics of a good is estimated. This demand in turn is used to calculate a weighted value of the characteristic mix of a product in a given market or at a given point in time. Such items as memory size, speed of addition, speed of multiplication, and access time to retrieve a random bit of imformation from memory are characteristics common to computers across the various generations. Brock [1] sought to proxy this technical progress by a price index and in that manner estimate a

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[2]See below, pp. 17-18.

technical progress coefficient overtime. A logarithmic estimation yielded

$$\log P = 1.54 - .237T \qquad R^2 = .99$$
$$(27.2)^{**}$$

Brock mentions this lies between values obtained by Knight [2]. Knight's observations were more elaborate, taking 148 models with thirty-two performance characteristics. Under the formula

$$\text{Cost} = \text{Performance}^{d} e^{-at}$$

Knight estimated the cost for both scientific performance and commercial performance. The respective values for the t.p. coefficient (.23 and .27) bracket Brock's value which was obtained from ten annual observations of price index and time. Subsequent estimations on a quarterly basis indicate the rate of technical progress has slowed. Computer scientists mention certain absolute barriers to further technical progress such as the speed of light and the size of an electron. Recent company reports mention further advances, but the data sets' termination means more recent estimations are impractical.

The nature of a limit price in the compúter industry is also elusive. Since IBM is the dominant firm, it would be the one to engage in limit pricing. The multitude of models, characteristics, prices, and products implies difficulty in defining a limit price. Use of hedonic price indices as above only approaches part of the problem. Delivery delays and market segmentation yield many opportunities for the potential entrant which are ill-addressed by a crude model of limit pricing. The above introduction (Chapter 1) detailed the legal discussion of limit pricing and the computer industry. It seems obvious (c.f. Gaskins, above) that a leading firm must have a cost advantage (i.e. $\bar{p} \geq c$) to limit price. Data on product cost of manufacture is not widely available. Even data base figures on cost of goods sold are aggregated to the multiproduct firm and therefore suspect.[3] Such imperfect data as are available must form the basis for estimating the limit price. Brock (op. cit.) postulates that limit price equals the costs of the nondominant firms. Brock averages excess profits of IBM (10%) and product differentiation and economies of scale differential favoring IBM (20%) to get a 15% spread between IBM's costs and prices (i.e., cost advantage). This assumes IBM has been limit pricing. Notwithstanding the decline in market share for IMB, Brock makes this assumption; i.e. that the IBM price is close to the limit price. A curious problem is that this limit pricing or quasi-

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[3]Disaggregated data are rendered suspect by accounting procedures.

limit pricing induced market penetration in the Brock schema. Perhaps the slow speed of market penetration reflects this "closeness" to the limit price. Brock also assumes declining limit price over time, an assumption common in the literature [3]. Another approach to the value for the limit price is to examine IBM's income reports. Broadly construed IBM's costs are costs of goods sold[4] and selling general and administrative expenses. These amount to some 75% of IBM's revenue. If this cost level is taken as IBM's cost, it is a floor for the limit price in the computer industry (c.f. Brock, above). Income reports of other leading computer companies indicate a consistently higher percentage of revenue for costs as described here. Accordingly a figure above 75% of IBM's market price can be used as a limit price estimation if IBM's price is in line with other companies' prices. Although IBM's mean price may be higher than other companies' mean price, a sign test indicates the IBM price is nearly as often below as above the competitors' price. Accordingly the limit price will be bounded by IBM's cost and price. Thus a price between 75% and 100% of IBM's price would deter rival market penetration in the computer market. A scarcity of data prevents

[4]This cost data collection is accounting data and therefore suspect.

narrowing of these bounds for the penetration deterring price.

Penetration has occurred in the computer industry at the expense of IBM's market share. While the overall trend is downward, IBM has gained market share in some years, lost in others.[5] IBM is so large that a sheer inertia seems to preserve market share in the GPC market. This size is maintained across the generations of products even though the innovations may not be initiated by IBM. No intragenerational pattern of market share change is observable. Any model predicting market penetration rivals appears contradicted in the long run by the tendency of IBM to regain market share. Data sets regarding market status can be gleaned from several sources. Brock (op. cit.) used data published in legal actions involving the computer industry. Other sources include the industry estimates made from the Computers and Automation monthly computer census [4].

This latter series can be modified according to the industry disaggregation employed. Yet another series can be obtained from Phister [5]. These series are collected in data Table 1. Omitted, though mentioned by Brock, is the IBM internal estimate of market share. Its

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[5]Accordingly Gaskins' model of entry (IBM market share loss) may not be appropriate.

monotonic decline stands in stark contrast to other series' rises and falls. A recent estimate of 67% for IBM market share (1978) compares with the 1956 figure of roughly 74%. Recent listings show fewer GPC companies with more models. The demise of RCA and GE computers left a gap filled by expansion of the companies acquiring their accounts and by the rise of the PCM's. A number of anti-trust suits have unbundled parts of IBM and changed its practices, but IBM's market share remains over 66% in the GPC market. Radical reorganization could effectively change that, but would require lengthy expensive governmental anti-trust action. This is an unlikely prospect with the current political climate for anti-trust.

Data in this industry, as in most areas of the private sector, is difficult to obtain. The basic computer variables are price and quantity. Due to technical change and inflation, these must be modified. When the required deflators are found, the original series may be adjusted to reflect quality characteristics and increasing machinery costs. External variables can be included to describe income effects, inventory effects, etc. The major stumbling block is the availability of basic data on price and quantity for the industry. Chow [7] constructed price and quantity series from internal IBM data and from trade publications (c.f. [4]

above). Brock added to these the legal figures labelled "Honeywell" in Table 1. Further annual data is found in Phister (op. cit.). However, a more detailed data series can be constructed from the Computers and Automation computer census. Running monthly from October 1962 to February 1969, this series shows number of installations, number of unfilled orders, and monthly lease price for all models of all major firms in the GPC market. Using the device of a presentative product, the median priced model can be selected in any month. A yearly summary in the same source gives quality data in the form of a comprehensive table of characteristics data. Since several firms are listed; aggregation of some to a representative firm, a rival for IBM, can be done. The aggregation chosen made a composite rival from Burroughs Corporation, Control Data Corp., Honeywell Inc., National Cash Register Corp., and Sperry Rand Corp. The other firms were omitted as too small, or as firms which left the industry (RCA, GE). A quarterly sampling of the series seemed most promising. Since the series deteriorated[6] in 1969, subsequent observations up to 1974 were deleted. Summing across a firm's

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[6] In 1969, a new firm was engaged to conduct the census. Its work proved unacceptable, and a new series measuring domestic and foreign installations was begun. This latter series exhibits little variation.

TABLE 1 -- IBM Market Share (%)

	Honeywell	Industry	Phister	Mod. Ind.	IDC
1955	56.1		79.2		
--	75.3	73.1	79.8		
--	78.5		79.4		
--	77.4	71.2	80.0		
--	74.5		81.8		
1960	71.6	70.7	80.0		
--	69.3		79.0		
--	70.0	70.4	80.8	83.1	
--	69.8	74.5	76.0	84.0	
--	68.3	72.5	70.0	84.0	
1965	65.3	66.7	65.6	79.2	
--	66.2	69.7	67.5	82.2	
--	68.1	74.3	68.9	86.9	
--		74.6	65.4		
--			67.4		
1970		70.6	67.5		
--		67.4	66.8		
--			66.6		
--			65.8		
--			65.8		
1975					
--					
--					
--					66.6
1980					

Sources: Honeywell in Brock (op. cit., p. 21), Industry (Ibid.), Phister (op. cit., p. 253). IDC ([6, p. 12, see also p. 8]). Modified Industry, vi.i.

TABLE 2 -- Competitor's Market Shares (%)

	Burroughs	Control Data	Honeywell	NCR	Sperry Rand	RCA	GE
1955	7.1			5.0			
--	6.6			4.1	8.6	.3	
--	6.9			2.6	7.4		
--	6.0			2.0	9.0		
--	5.2			1.1	9.7	.5	
1960	4.0		.8	1.5	10.0	1.0	.2
--	2.7	.4	.8	2.1	10.5	2.0	1.4
--	2.2	.4	1.0	4.0	7.2	2.5	1.3
--	2.6	.6	1.1	4.9	9.1	3.4	1.8
--	3.1	.7	1.9	5.3	14.2	3.2	1.4
1965	3.3	1.1	3.5	5.9	15.3	3.0	1.9
--	2.9	1.2	4.4	5.4	12.7	2.7	2.8
--	3.2	1.1	5.2	5.3	10.1	2.5	3.2
--	3.2	1.2	5.4	8.3	9.4	2.8	3.5
--	3.5	1.1	5.2	6.6	9.6	2.4	3.0
1970	3.5	1.1		7.6	9.6	2.2	
--	4.1	.9	6.2	7.0	9.4	1.9	2.2
--	3.9	.9	7.0	6.9	8.7	1.9	2.4
--	4.5	.8	6.8	7.1	8.0	1.5	2.7
--	5.0	.8	6.5	7.8	6.9	1.3	2.5
1975							
--							
--							
--	5.3	2.4	5.7	2.2	6.6	--	--
--							
1980							

Sources: 1978 data IDC (op. cit., p. 12). All other years Phister (op. cit., p. 253).

models yields a number for stock of installations. For IBM, its stock is this sum, its price the median price. The rival series was constructed by summing company stocks and weighting sums of median prices. This weighting scheme was also applied to the construction of hedonic price indices to correct for quality change (see pages 59-61). A separate estimation for IBM proved fruitless -- there the hedonic price index was rejected on economic grounds. The stock series covers some twenty quarters. Further computer company data comes from Moody's [8] on an annual basis. This material, supplemented in Phister (op. cit.), gives information on costs and inventories. As it is firm-specific, "rival" inventory can be constructed. Similarly selling, general, and administrative expense can be totalled. Among these sources is a variety of computer specific data.

Other data refer to external effects incumbent upon the computer industry. A price deflator can be used to correct for inflation. Of those available, the machinery and equipment component of the wholesale price index most closely approximated the computer industry. An income effect was proxied by a series in final sales of the U.S. economy sampled quarterly in the GNP accounts. Advertising data is available monthly in an aggregated form. Total advertising and business periodicals advertising are both compiled in the McCann-Ericson advertising index. Another series involved capacity utilization in manufacturing industries. This series constituted part of an inventory proxy. Another inventory proxy involved rival inventory and change of production. Series in capital stock for the manufacturing sector and in research and development expenditures by the Department of Defense were sought in vain. The data including the inventory proxies are listed in Table 3.

By contrast, the data series in Chow and Brock are much more limited. Chow uses an annual series in rental values for his quantity. His hedonic price index substitutes speed of multiplication for the often-used speed of addition as a computer characteristic. Since he studied new models, a scarcity of data caused him to pool observations over several years. His price series, from similar sources (IBM data, Weik reports, Computers and Automation), is deflated by the GNP deflator. On a quarterly basis a different deflator should be used to avoid interference from agricultural variations. His income proxy data proved unavailing; he used series in real GNP. Brock added time to his data set and subtracted GNP and quantity logged. He did, however, include the legal data on market share of IBM. Chow is able to explain a changing demand curve, and Brock adds supply

TABLE 3 -- DATA

PROXY 1 R1	PROXY 2 R2	X PROD	Market Price AP	Q PROD	PROD TOTAL
-	-	-	30724.8	-	-
-	-37316	160	27410.0	1412	1572
5113.80	-38604	583	25501.4	918	1501
3071.40	-28577	118	24475.2	780	898
5035.60	-38163	507	17744.8	1167	1674
4181.80	-38282	562	17791.7	1044	1606
4678.60	-39380	833	17780.8	802	1635
3905.80	-40942	768	15208.0	942	1710
3523.00	-40256	290	15315.0	1346	1636
6485.40	-40388	1100	18426.2	310	1410
3718.50	-38407	707	18410.2	849	1556
5346.90	-38826	1022	18051.8	1133	2155
4670.25	-39393	338	18293.4	1290	1628
6353.85	-39002	386	20976.9	447	833
7145.05	-37791	778	20942.2	2858	3636
6070.95	-35846	703	25233.1	2754	3457
8181.90	-34870	1162	25372.2	3534	4696
8340.60	-32455	1690	15456.3	8841	10531
5626.50	-39310	810	15115.1	1900	2710
6960.50	-39486	510	15360.2	2500	3010

TABLE 3 -- DATA, Continued

Q STOCK	X STOCK	X PRICE	WPI M&E	QUALITY INDEX	Q PRICE
7273	2627	23867	92	15	14000
8685	2787	21130	92	16	14000
9603	3370	17384	92	16	14000
10383	3488	15370	92	16	14000
11550	3995	9406	93	17	12000
12594	4557	9505	93	17	12000
13396	5390	9482	93	17	12000
14338	6158	13079	94	24	14000
15684	6448	13401	94	24	14000
15994	7548	13426	94	24	18000
16843	8255	13378	94	24	18000
17976	9277	15164	95	25	18000
19266	9615	16539	96	25	18000
19713	10001	16592	97	25	22000
22571	10779	17776	99	25	22000
25325	11482	14555	100	20	22600
28859	12644	14926	100	20	22600
37700	14334	15343	102	29	19550
39600	15144	15166	104	29	19550
42100	15654	16152	104	29	19550

(via entry) to equilibrate (close) the system. More detailed data is available and can be used to estimate key parameters in the computer industry.

It is worthwhile to examine the construction of and trends in the data of Table 3. Specifically, for an application of Gaskins' model, basic variables such as rival market penetration (RMP) and market price are required. First, rival market penetration equation estimation from Gaskins means RMP = $K(p-\bar{p})$. This is a simplified form of a relation which may include time and other variables. The price variable can be obtained by finding a weighted average of IBM price and rival price. Recall these prices have already been adjusted for inflation and quality change. Such a market price can also be used in the demand estimation, preserving the link between the equations Gaskins postulated. Rival market penetration is a more difficult nut to crack. Customary measures of entry provide very few observations and the timing of entry frequently vitiates even those few observations. Accordingly rival market penetration will be employed. This follows the suggestion of Bain (introduction, op. cit.). The construction of this proxy is somewhat involved. First one must observe that the market in used computers is significant and that computers can be resold (re-leased to another user) when more modern models are leased or bought. Thus the first difference of the computer stock series is considered production of machines. To proxy market penetration, the production series for the rival is first differenced. This is a short run series and is explicitly required to be positive. A negative term here is stored for separate estimation.

Similarly the demand analysis stems from the market price and production variables. These are constructed as in the entry estimation. Production is totalled for the rival and IBM. This is done to retain the aforementioned link. The two proxies are constructed to contain inventory and other effects. R_1 contains inventory for the composite firm and production change for the same. R_2 contains the income term (final sales), a lagged IBM stock term reflecting stock adjustment and equilibrium adjustment, and the capacity term reflecting a pure inventory effect. The second proxy stemmed in part from the high correlation between the income term and the lagged stock term. Both were recommended by Chow, but the high correlation made their separate inclusion infeasible. The first proxy is similarly motivated.

With the data in appropriate form, one may proceed to specify and estimate the model. RMP can be broadly stated as

$$RMP = f(p, C, Q_{-1}, \ , \ , \ ,) \qquad (1\text{-}1)$$

where P is price, C cost, Q_{-1} the stock of IBM lagged, and other variables may be included. For computational purposes, equation (1-1) should be linearized so that its form becomes

$$RMP = A + bP + dC + eQ_{-1} + \ . \ . \ . \ + u \qquad (1\text{-}2)$$

Further simplification renders the estimation more comparable to Gaskins' work, thus from (1-2) is written

$$RMP = k(p - \bar{p}) + v \qquad (1\text{-}3)$$

Observe that the constant (A) and the error term (u) have been rearranged in equation (1-3). Next it has been suggested that a simultaneous equations model estimation be used for entry [10].

$$\begin{aligned} RMP^d &= a + bP + cY + u \\ RMP^s &= d + eP \quad + u \end{aligned} \qquad (1\text{-}4)$$

Equation (1-3) and system (1-4) were estimated with results as follows. For (1-3), the estimated equation was

$$\underset{(8.00)^{**}}{RMP = .019P} \qquad R^2 = .93, \quad n = 6 \qquad (1\text{-}3e)$$

Different specifications with more data points lowered the R^2 and the t- value. Similarly inclusion of more explanatory variables lowered the t- value. However both augmentations lowered the coefficient of P. Thus a larger data set or more complete set of explanatory variables might lower the value of the response coefficient even further. It is required that this coefficient be positive. The ex post optimal value in simulations was .004, which confirms the value of the lowering trend. Also, the other approach (simultaneous equations modeling) was employed. As is indicated, the instrumental variable used was the income proxy, final sales. The resulting estimate was

$$\begin{aligned} RMP &= 142+.011P-.011\bar{P} \\ &= 142+.011P-.011(13000) \\ &= 142+.011P-142 \\ &= .011P \end{aligned} \tag{1-4e}$$

Other market penetration terms were used and other instruments employed. The lowest positive response coefficient obtained was .0008. The various estimations bracket the simulation value. This can be compared with other economists' estimations. Chow did not consider RMP, but Brock estimated it via integral-equations. Values from his model range from .01 to -.005 on a quarterly basis. Since

TABLE 4 -- RMP

Data	Sample	Dependent Variable
Honeywell Brock	values for 1963 and 1965 (69.8,65.3)	marketshare
Quarterly (OLS)	I n=6	RMP
Quarterly (OLS)	II n=7	RMP
Quarterly (OLS)	III n=6	RMP
Quarterly (IV)	IV	RMP
Quarterly (IV)	V	RMP2

TABLE 4 -- RMP, Continued

Constant	Causal Variable	R^2	DW
N/A	entry K_o=.03	N/A	N/A
NOINT	price .019 (8.00)**	.93	N/A
NOINT	price .017 (7.47)**	.90	N/A
NOINT	price, expense .014, .054 (2.07)** (0.76)**	.94	N/A
142	net price .011	N/A	N/A
1339	price .0008	N/A	N/A

he gave only one estimate, the reader can not assume he rejects (or accepts, for that matter) negative values for the response coefficient. For ready comparison, the estimations are exhibited in Table 4.

Next the demand relation must be estimated. In its most general form, the industry demand may be given as

$$\text{Demand} = g(P, Y, \ldots) \quad (2\text{-}1)$$

This can be linearized and simplified to

$$\text{Demand} = A - bP + cY + \ldots + u \quad (2\text{-}2)$$

To preserve the Gaskins' formulation of demand disaggregated to dominant and rival demands, such a decomposition will be done. Further the same market price is used to link the two estimations. The resulting demand form[2] is

$$Q + X = Z - bP + cY + \ldots + u \quad (2\text{-}3)$$

However for ready comparison with Gaskins form, the income effect can be included with inventory effects for a proxy. As above correlation coefficients caused the original

[2] This relation is estimated by single equation (OLS) estimation. Simultaneous equations methods produced a bad fit.

use of inventory proxies. Two different proxies were used, one with an intercept and one without. Thus

$$\hat{(Q+X)} = -.15P + 1.00R_1 \qquad R^2 = .80$$
$$(-2.38)^{**}(4.62)^{**}$$

$$\hat{(Q+X)} = -.26P + 1.06R_2 + 48098 \qquad R^2 = .88$$
$$(-5.33)^{**}(19.95)^{**}(11.67)^{**} \qquad n = 18 \qquad (2\text{-}53)$$

Related estimates for comparison are those of Chow and Brock.

For Chow, $\log (P^*.(Q+X) = -.3637 \log P - .2526 \log S_{Q+X} + 2.950$
$(.1726) \qquad (.0739)$

His notation is changed to conform to that above. Accordingly Q+X, the stock difference is used for his Y_t/Y_{t-1} , S for his Y_{t-1}, P* to indicate value, and P for market price. In another estimation, his income term entered with the wrong sign. Chow's proxy to change this involved the price term. Thus while the numbers are otherwise acceptable, there is difficulty in the interpretation, especially with the proxy and the dependent variable. Brock employed the Chow data in a different dynamic estimation. Originally he proposed the equation

$$Se^{gt} = Ap^b e^{gt} u \qquad (B\text{-}1)$$

which failed during estimation since p and t were highly correlated. Next

$$Sp^{-b}e^{gt} = Ae^{gt}$$

or in a more useful form

$$\tilde{S}p^{-b} = Ae^{gt}$$

which is estimated as

$$\log \tilde{S} - b \log P = 9.225 + .147t \qquad R^2 = .82 \qquad \text{(B-2e)}$$
$$(6.08)^{**}$$

The parameter b is taken from Chow (it is Chow's estimate of the price elasticity of demand). As in Chow the dependent variable is difficult to interpret, and the t-value for the intercept is omitted. No Durbin-Watson statistic is given by Brock. Also he excludes income and stock effects proposed by Chow, and inventory effects suggested above. This latter seems crucial in light of Brock's long run implications drawn from his model. Brock's estimation of the technical progress coefficient is not repeated on a quarterly basis because the R^2 was so low. From his market share equation and estimated parameters, Brock gets a simple equation for the discount rate. He obtains the high value of .2733, consonant with a major firm in a rapidly growing industry. By contrast others (e.g. Gaskins) do not estimate r. A quarterly

TABLE 5 -- Demand

Data	Sample	Dependent Variable
Chow	I n=ii	log S
Chow Brock Model	II n=10	logS+1.44logP
Quarterly	I n=18	X+Q
Quarterly	II n=19	X+Q

TABLE 5 - Demand, Continued

Constant	Causal Variables	R^2	DW
2.950	price, S_{-1} (-.3637), (-.2526) (-2.11)** (-3.42)**	.83	1.77
9.225	time .147 (6.08)**	.82	N/A
NOINT	P, RI -.15, 1.00 (-2.38)**(4.62)**	.80	2.04
48098 (11.67)**	P, R^2 -.26, 1.06 (-5.33)**(10.95)**	.88	1.72

estimation might very well ignore discounting.

The estimations above[7] have many interesting implications. The quarterly study may be viewed as constructing a series of short run steady state models. A long run study would make more explicit reference to inventory effects. However such a long run study might prove misleading due to the market growth and innovation in the computer industry. The estimates except for Chow's are elementary applications of Gaskins' theory. Accordingly the implications of Gaskins' theory can be traced through the estimations here. As in Brock, myopic profit maximizing is non-optimal. Also the estimates imply an elastic demand for computers. Recall Chow's value of 1.44 which Brock accepted for the price elasticity of demand. The quarterly estimations above yielded values for the price elasticity of 1.20 and 2.06. These are obtained by multiplying the demand slopes by the quotient of mean price divided by mean production.[8] Other facets of the estimation also have economically and econometrically acceptable effects. The demand curves slope down in all estimations; further, RMP (which may be

[7]Only the best estimations are shown. The many bad runs have been omitted.

[8]Demand slopes are reported in Table 5. The mean price and mean production are computed from the data set in Table 3.

viewed as change in supply) has a positive price effect. Positive intercepts are gained for the demand curve. Econometrically, the R^2 values are high, t- statistics significant at the five (or sometimes the one) percent level. The Durbin-Watson d- statistic is acceptable, and the F- statistic also indicates the value of the estimation. This is true of the quarterly studies for which statistics are available. Likewise the annual studies of Chow and Brock exhibited acceptable descriptive statistics in so far as they were reported. Specific comparisons can highlight these matters.

The earliest computer study considered here is that of Chow (op. cit.). A comparison with the proposed model indicates the partial nature of Chow's model,[9] but also shows its strengths. Chow pooled data on new computer models from 1960-1965 to estimate the coefficients for hedonic price index construction (see Table 6). His 1959 figures would also give proper signs and descriptive statistics. The quarterly estimation made use of similar annual data on characteristics. Although more models were available in the chosen year, the two sets of coefficients were inferior to those of Chow. For IBM, the signs were correct, but the low R^2 (.46) and a

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[9]Chow does not estimate supply.

TABLE 6 - Estimation for
Hedonic Price Index, Continued

Constant	Causal Variables	R^2	D-W
2.489	MU, MS, AS -.21, .36, -.13 (-5.78)**(9.02)**(-3.76)**	.993	N/A
-.1045	MU, MS, AS -.0654, .5793, -.1406 (-2.30)**(16.36)**(-4.80)**	.91	N/A
4.27 (3.87)**	logs of MU, MS, AS -.28, .09, -.10 (2.98)** (0.35)**(-1.15)**	.58	1.06
2.04 (2.29)**	log MU, log MS, log AS -.09, .40, .04 (-.95)** (2.24)**(0.59)**	.46	1.99

Table 6 - Estimation for Hedonic Price Index

Data	Sample	Dependent Variable
Chow	1959 n=10	rental
Chow	1960-1965 pooled n=82	rental
IBM Quarterly	1967 n=20	log P
Others Quarterly	1967 n=38	log P

wrong sign for the access speed variable suggest problems. Nevertheless the hedonic price index constructed for the rival was acceptable; that for IBM was not. Thus the logarithmic estimation for the rival characteristic value was used to construct the hedonic price index used to deflate both series for quality change. The Chow demand estimation features an R^2 of .83 with proper signs (significant) on the price, stock, and intercept terms. The quarterly study similarly had acceptable statistics for intercept (where used), price, and inventory terms. Chow lost the income term due to a perverse sign; the quarterly study excluded it due to a correlation problem. The second generation takeover caused a large error (underestimation) in Chow's estimate for 1961. A similar phenomenon (but only for the study omitting the intercept) could be found in the quarterly study when the third generation impacts upon the industry. The quarterly estimate with intercept has its largest (underestimation) error when the second computer generation ends. The models will not be compared further because Chow omits the supply side of the market.

Brock takes up the supply side and thus his model can be compared more fully to the quarterly model. His supply work consists of an integral equation which describes market penetration. The source of data for this equation is the legal series of IBM market share data. An arbitrary data selection process vitiates Brock's procedure. Since Brock discards all but two data points, selection of those two should be carefully justified. Choosing high and low points in a product generation is not acceptable for an intergenerational study, especially when the suggested maximum is not a maximum at all. The alternative procedure of defining market penetration on a composite company basis has much to recommend it. First such market penetration is definite and meaningful: an acceleration of fifty machines per quarter-squared can be understood. The analogy with physics is clear. Second, this estimation depends directly on price, in a clear parallel with Gaskins. Third, such an estimation can be subjected to the customary economic and econometric tests. Fourth, any restrictions are made and justified a priori. Thus positive price effects, high R^2, significant t-statistics, positive RMP values can all be required of the quarterly study. By contrast the integral equation quickly yielded an exit coefficient (not mentioned by Brock) for the next two points. This would notably alter his market share prediction, perhaps indicating a negative market share for IBM! To specify, his market share equation is

$$\hat{S} = \frac{-K_o(\bar{P}_o - C_o)}{-2ba - a(b-1)b\frac{(1-C_o)}{\bar{P}_o} + (g-r-a)\frac{(1+b-bC_o)}{\bar{P}_o} + \frac{abC_o}{\bar{P}_o}} \quad \text{(B-14)}$$

Since $\bar{P}_o > C_o$, $K_o > 0$ by Brock's estimate, the numerator is negative. Thus if the market share $\hat{S} = 1-\hat{x}$ is positive, the denominator must also be negative. Yet a reasonable selection from the data (market share at the start of the third-integrated circuit-generation which IBM neither invented nor caused) yields just such an exit coefficient. This, by the last equation, implies a negative market share.

$$\frac{+}{-} = -$$

The analysis and comparison of Brock's other estimations are similar, if less striking. The concentration on technical progress is accomplished at the expense of studying income effects, stock effect, inventory effects, etc. Time explains increasing value of computer installation rather well in the Brock model. Similarly time explains price. While the stated concerns are, respectively, growth and technical progress, theoretical time seems an odd proxy by which to study those phenomena. The implication of years passing is steady growth in demand and steady decline in price via technical progress.

Accordingly economic recessions or a slowing of innovations inter alia would imply ever larger errors in the Brock framework. Similarly a burst of inventions or a successful advertising blitz is outside his model. By contrast, the quarterly model includes proxies which cover such items as inventory, income, and the like. Brock's model consistently understates demand, which the quarterly model does not. This is a problem not found in Chow's estimation employing the same (Chow) data Brock uses. The predicted price level understates the actual (Chow's price index) through most of the series. These errors may contribute to the consistent overestimate of market share found in Brock's model.[11]

A final comparison is to a more recent theoretical contribution of Rifkin and Sengupta [9]. This is appropriate for discussion because of the simulation studies it contains. The simulation is again based on a theoretical formulation stemming from Gaskins. While Brock added technical progress to the Gaskins growth model. Rifkin and Sengupta re-state Gaskins with quantity as a control, risk aversion, and stability analysis. Several values are used for the demand slope, the entry coefficient, the discount rate, and the growth rate. A large response

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[11]This market share problem with Brock's work is due to a lack of econometric estimation.

coefficient implies loss of market share, the more so with higher growth rate, lower discount rate, and an algebraically larger demand slope. Indeed the response coefficient outweighs in importance market growth as a key parameter. The model is also quite sensitive to the initial values and to relations between the discount rate and the growth rate. Since the quarterly model uses no discount rate, comparisons are more difficult with this model than with Chow's. Likewise growth is explicit in the Rifkin and Sengupta formulation implicit in the quarterly model. The key nature of the response coefficient was confirmed in prototype models leading to the quarterly models above. The sensitivity of the Rifkin and Sengupta model to the demand slope is unclear. However an increase in the absolute value of the demand slope increases price elasticity of demand as in the quarterly model. The Rifkin and Sengupta model is constrained to be stable via the transversality condition. This could well alter the generated simulation paths. The quarterly model deals in discrete time and employs a bequest term. This finite analog to a transversality condition can be set equal zero to avoid such implications. If it were known a priori that the transversality condition could effect the model, this could be expressed explicitly in the form of a different set of equations which would lead to different simulation paths. The use of scaling and discounting is required to stabilize the Rifkin and Sengupta model in the long run. As the quarterly model is avowedly a short run study, scaling and discounting are not required for it. One advantage of the Rifkin and Sengupta model, though, is the ease of interpretation gained by the explicit growth formulation. Implications of a growing market can be studied much more readily when the nature of the growth is immediately apparent. It would be a valuable exercise to apply coefficients of the quarterly (or another) model to the Rifkin and Sengupta model to see if its implications are confirmed.

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10. Thanks are due to H.E. Frech III for suggesting this procedure.

APPLICATION VIA DYNAMIC PROGRAMMING

Dynamic programming can be used to obtain an optimal solution in the dynamic limit pricing scenario. The procedure is to set up a series of maximizations. Thus it is required to

$$\max_{q_t} V_t = \max (\pi_t + V_{t+1})$$

and

$$\max_{q_{t-1}} V_{t-1} = \max_{q_{t-1}} (\pi_{t-1} + V_t)$$

and

$$\max_{q_{t-2}} V_{t-2} = \max_{q_{t-2}} (\pi_{t-2} + V_{t-1})$$

and so forth. Observe the profit function

$$\pi_t = (Y_t - bQ_t - bX_t - C)Q_t \qquad \text{(Pi-1)}$$

is a linearized form comparable with equation (2-3) given above. If this series is assumed finite and discrete, V_{t+1} can be viewed as a bequest term.[1] Therefore let $V_{t+1} = -dX_{t+1}$. A discrete version of Gaskins' entry (Pi-2) equation completes the model.

$$X_{t+1} - X_t = k(p_t - \bar{P}) \qquad \text{(Pi-3)}$$

[1]This bequest shows the state of the rival in period T+1 and is the finite analog of the transversality

The maximizations can be solved manually or by a computer program. For comparison purposes, the bequest coefficient (d) can be set equal zero. Observe that the maximizations here are amenable to simple techniques from differential calculus. Thus

$$\max_{q_t} V_t = \max_{q_t} (Y_t - bQ_t - C)Q_t - dX_{t+1} = Y_t - bX_t - C - 2bQ_t - \frac{d}{dQ_t}(dX_{t+1}) \qquad \text{(Pi-4)}$$

The latter term is zero when d=0 and the resulting equation is Q_t is $Q_t = \frac{Y_t - bX_t - C}{2b}$ (Pi-5)

The second problem in Q_{t-1} explicitly involves the entry equation and is more complex. The resulting expression is

$$Q_{t-1} = \frac{Y_{t-1} - bX_{t-1} - C + kQ_t b^2}{2b} \qquad \text{(Pi-6)}$$

again with d=0.

To generalize the dynamic programming analysis, these equations (from the d.p.) may be used in simulation studies. By using these equations in a stochastic setting, one obtains further information on the robustness of the basic system. Just as in the dynamic programming study,

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condition.

parametric values estimated above can be substituted into the appropriate equations. A FORTRAN simulation postulated imperfect information for the dominant firm. Thus rival market penetration may be perceived wrongly. Further a feasibility check was inserted via an asset structure term. Accordingly a plunge to zero assets would terminate a simulation run. Similarly prices and quantities were required non-negative. The FORTRAN subroutine GGUBFS was used to generate the random term in rival production. Another simulation program yields a rapid validation procedure for the dynamic programming version of the model. DYNAMO works in continuous time thus approximating the discrete time model above. Stocks and flows are clearly distinguished and initial conditions can be specified. Since the time between samplings can be minimized, the approximations should be rather accurate.

The final simulation was the discrete program package SIMSCRIPT II.5. As it was based on the FORTRAN program, similar feasibility checks are used. Again the rival production may be estimated with error by the dominant firm. In spite of the procedures in common, the results of the two packages are notably different. The difference could be even greater if the SIMSCRIPT program were extended to the more complex fifth level estimation. This will be discussed in more detail in the chapter on extension. Likewise the simulations will be more fully delineated in the following chapter. Their purpose is to validate the model and its use of dynamic programming.

A dynamic programming algorithm is merely one of several devices to validate a limit pricing analysis. Brock[1] uses numerical analysis to obtain the time paths of price and quantity. The initial values stem from the IBM market share series and the entry equation (also in 1956). Thus initial rival market share is .247 (1 - .753, the .753 being IBM's market share in 1956). Assuming limit pricing by IBM, the market price index for 1956 is set equal to the limit price index then prevailing. This system is unstable because any shock (such as a technological innovation) causes divergence from the unique optimal path. Also his system is sensitive to initial conditions as well as time rates of change. The chosen value p = 2.593 exceeds slightly the limit price value of 2.53. Further the predicted path for market share of IBM declines monotonically to 67.8%. Another specification set the growth rate for computer demand equal to zero. This suggests a much lower market share. However with the current rate of growth, Brock's model consistently overestimates IBM's market share. Also his model underestimates market price level in index form. Demand rises at about the same rate as Chow's quantity index. The initial quantity lies below that of the actual)Chow) series.

A more recent validation of a limit pricing analysis is that of Rifkin and Sengupta [2]. No actual series is provided for comparison, but the authors' work stems from a formal stability analysis. Simulation paths are chosen stable a priori. As with Brock, initial conditions are very important. Here they determine time to study state convergence. Scaling and discounting are of similar importance. A high response coefficient causes a decline in the dominant firm's quantity in these simulations. When this coefficient (Gaskins' "k") is lowered from .25 to .1 or .01, dominant firm quantity rises. The latter value also causes the competitor's quantity to fall in this scaled time path. An algebraic increases in the demand slope accentuates movement of dominant firm quantity. While the discount rates and growth rates were reasonable, they were exogenous to the model. Also, the shadow price is adjusted for sign, but not the dominant firm's quantity. In this simulation it is difficult to rationalize a negative quantity. Switching from seller to buyer seems odd when the competitors' growth is not greatly increased.

The use of dynamic programming (d.p.) vitiates these problems. Any knife-edge problem as Brock envisions can be carefully scrutinized. An exogenous shock could be explicitly handled in the intercept term. This flexibility promotes a closer fit: a series need not be routinely overestimated. As with the simulations above, basic variables prices, quantities, cost and demand intercept were assumed positive. The "backwardization" nature of dynamic programming requires the specification of a terminal condition for rival production. This, like the parameters in the model, can be subjected to sensitivity analysis. The parameters include the bequest coefficient, the demand intercept, the response coefficient, limit price, and cost. Further, the demand slope should be varied in this study. The details of this procedure can be set forth for the interested reader.[2] (Likely values as from the above computer estimation can be used in the algorithm. The OAS routine (see Appendix 1) permits specification of initial values, terminal values, and incremental values for each of the above except the demand slope. The initial set of values is employed for a fifty period count or until termination via convergence (repeating values on prices within $1.01). One of the parameters is incremented and the procedure repeats. When the terminal value is reached, the second

[2]See appendix program, "Optimal Analytic Solution" for details.

parameter is incremented. Each run opens with the terminal equation set, then continues with the loop. The loop could be repeated indefinitely but is generally stopped at fifty periods. Inside the terminal set and the loop are the dynamic programming equations detaching the relations between quantity and price, profit and price, etc.) To confirm the validity of the routine manual calculation is employed. A data set at the end of the routine can be changed to allow longer runs, closer convergence, more rival production, or different parametric values. In the short run, a discount rate (or growth rate) need not be employed. Growth may be included in the intercept manually. The d.p. routine explicitly assumes profit maximization subject to entry and shows this interdependence in its equations. The roots of the other validations seem more arcane as they stem more from complicated differential equations which are not amenable to analytic solution.

Several cases of optimal solutions can be found using various parametric values obtained above (see Tables 1-5). Key parameters varied include the demand slope, the response coefficient, the bequest coefficient, and the terminal value for rival quantity. It will be noted that the market share compares well with that of Gaskins [3] in his proposed model.

TABLE 1

Q	X	P	T
2937	500	31561	20
3028	314	30869	
3022	135	30799	
2995	0	30617	
2902	0	30006	
2712	0	30063	15
2624	0	29476	
2538	0	28904	
2454	0	28344	
2372	0	27798	
2292	0	27265	10
2214	0	26746	
2138	0	26240	
2064	0	25747	
1992	0	25264	
1921	0	24794	5
1852	0	24335	
1785	0	23889	
1720	0	23452	
1656	0	23026	1

Parameters: $k = .01$, $b = -.15$, $d = 0$, $x_n = 500$

TABLE 2

Q	X	P	T
2937	500	31561	20
2857	481	30897	
2768	463	30309	
2681	446	29734	
2597	429	29179	
2514	413	28630	15
2434	397	28100	
2355	382	27578	
2278	367	27072	
2203	353	26572	
2130	339	26089	10
2058	326	25613	
1988	313	25153	
1920	301	24700	
1853	289	24260	
1788	278	23827	5
1724	267	23407	
1662	257	22994	
1602	247	22593	
1542	237	22203	1

Parameters: $k = .001$, $b = -.15$, $d = 0$, $X_n = 500$

TABLE 3

Q	X	P	T
1353	500	17208	20
1459	458	17415	
1542	414	17717	
1625	367	18026	
1710	317	18340	
1797	264	18660	15
1885	207	18989	
1975	147	19320	
2066	84	19656	
2158	17	20002	
2226	0	20247	10
2285	0	20462	
2343	0	20678	
2401	0	20892	
2459	0	21106	
2517	0	21320	5
2574	0	21534	
2632	0	21746	
2689	0	21956	
2746	0	22170	

Parameters: $k = .01$, $b = .26$, $d = 0$, $X_n = 500$

TABLE 4

Q	X	P	T
2937	500	31561	20
3028	314	30869	
3022	135	30799	
2995	0	30617	
2902	0	30006	
2712	0	30063	15
2624	0	29476	
2538	0	28904	
2454	0	28344	
2372	0	27798	
2292	0	27265	10
2214	0	26746	
2138	0	26240	
2064	0	25747	
1992	0	25264	
1921	0	24794	5
1852	0	24335	
1785	0	23889	
1720	0	23452	
1656	0	23026	

Parameters: $k = .01$, $b = -.15$, $d = -.1$, $X_n = 500$

TABLE 5

Q	X	P	T
3292	0	32524	20
3190	0	31884	
3089	0	31256	
2991	0	30644	
2895	0	30049	
2802	0	29466	15
2711	0	28900	
2622	0	28346	
2535	0	27805	
2450	0	27277	
2368	0	26762	10
2287	0	26260	
2209	0	25771	
2132	0	25294	
2057	0	24828	
1984	0	24374	5
1913	0	23929	
1844	0	23498	
1775	0	23085	
1656	0	23024	1

Parameters: $k = .01$, $b = -.15$, $d = 0$, $X_n = 0$

Some confirmation of this optimal model is thereby found. Gaskins has a similar growth rate (8%) and ratio of market price to limit price (1.1) which lead to a market share of .82 (C.F. Chapter 3, Table 1, model indicated column). Yet another validation of these dynamic profiles is the stability analysis.

To proceed with the stability analysis, recall the basic model:

$$H = (Y_t - bX_t - bQ_t - C)Q_t + Z_t k(Y_t - bX_t - bQ_t - \bar{P}). \qquad (S-1)$$

For convenience the subscripts will be dropped in the ensuing equations. The necessary conditions for a maximum are similar to those of Gaskins.

$$\dot{Z} = -\frac{\partial H}{\partial X} = bQ + bkZ, \quad \lim Z(t) = 0$$

$$\dot{X} = \frac{\partial H}{\partial Z} = k(Y - bQ - bx - \bar{p}), \quad x(0) = X_o \qquad (S-2)$$

$$0 = \frac{\partial H}{\partial Q} = Y - bx - C - 2bQ - bkZ.$$

These necessary conditions lead to a system in x and Z.

$$X = bkx - bkZ + \gamma_1$$

$$(S-3)$$

$$Z = bkx + bkZ + \gamma_2$$

where the γ's are autonomous terms. This system is singular. Bell and Jacobson [4] discuss procedures for breaking singularity. Reducing the dimension space seems unavailing, so the model will be reformulated. The long run models (Gaskins, Brock, Rifkin and Sengupta) used growth and discounting. Instead of this remedy, a short run study demands a different method. Suppose, as in learning theory, the dominant firm lacks perfect knowledge of the competitor's simultaneous production. Then the dominant firm might estimate entry using perceived entry and rival stock. Accordingly set

$$\hat{\text{Entry}} = A_1 \text{ Entry} + A_2 X \quad \text{(S-4)}$$

and normalize to the form

$$\hat{\text{Entry}} = \text{Entry} + \alpha X, \; \alpha = A_2, A. \quad \text{(S-4A)}$$

This changes the Hamiltonian to

$$H = (Y_t - bX_t - bQ_t - C)Q_t + Z_t k(Y_t - bX_t - bQ_t - \bar{p}) - Z\alpha X. \quad \text{(S-1-A)}$$

Again write the necessary conditions

$$\dot{Z} = -\frac{\partial H}{\partial X} = bQ + bkZ + \alpha Z, \; \lim_{t\to\infty} Z(t) = 0$$

$$\dot{X} = \frac{\partial H}{\partial Z} = k(Y - bX - bQ - \bar{p}) - \alpha X, \; x(0) = X_o \quad \text{(S-2-A)}$$

$$0 = \frac{\partial H}{\partial Q} + Y - bX - bZ - C - 2bQ.$$

Reduction to a system in X and Q gives

$$\dot{X} = -bkx - \alpha x - bkQ + Y_1 \quad \text{(S-3-A)}$$

$$\dot{Q} = bkx \pm \alpha x + bkQ + Q + \gamma_2$$

The characteristic equation is then

$$X^2 - \alpha^2 - bk\alpha = 0$$

which is nonsingular unless $\alpha = 0$ or $\alpha = -bk$. For these to happen stock must be ignored in estimating entry, or the dominant firm must assume infinite rival entry, or ($\alpha = -bk$) which also seems unlikely. As entry proceeds and rival stock increases, this relation ($\alpha = -bk$) might occur once. Even with entry and exit, it still seems improbable that an occasional singularity would disturb the firm's planners unduly. A simple decision rule would suffice for the quarter when the singular system yielded no information. Aside from the singularity the system is robust. The characteristic roots are opposite in sign ($\pm\sqrt{\alpha^2 + bk\alpha}$. As long as customary economic laws are obeyed (response coefficient positive, demand slopes down), the roots will not be imaginary as long as $\alpha > 0$. This in turn requires that the dominant firm estimate entry with some minimal acuity and that the stock effect on entry be positive. Violation of these rules is doubtful, but theoretically possible. If IBM estimates exit when entry is strongly indicated, or assumes larger rival installation stock means less entry; then and only then could these restrictions be violated in the computer industry. As in the d.p. runs, the model yields acceptable values and series. The effect of changing the demand slope is striking. The response coefficient at such small values affects the rival more than the entrant. This is also true of the terminal rival production. Large bequest coefficient values would be required to change the dominant firm's planning significantly. The series constructed by dynamic programming have considerably smaller variances than the actual series. In a risk averse firm this accentuates the optimal quality of the d.p. figures. This will be examined in greater detail in the next chapter on simulation.

APPENDIX 1

```
Early d.p. routine
LEONARD JOB (4247, Leonard, 2, 8,,3060,2),'Q'
 EXEC PLC,TIME=(1,30)
O DD SYSOUT=A
SYSIN DD *
PLC PAGES=500
*     OPTIMAL ANALYTIC SOLUTION
*-------------------------------------------------------*/
*PROGRAM:CREATED 10/31/78*/
*AUTHOR:RADFORD A. DAVIS*/
*PURPOSE:TO PROVIDE AN OPTIMAL ANALYTIC SOLUTION TO A
* DYNAMIC*/
*PROGRAMMING PROBLEM IN ECONOMICS  */
*-------------------------------------------------------*/
  /*-------------------------------------------------------*/
  /*    THIS BLOCK DEFINES THE INPUT VARIABLES.  THEY      */
  /* ARE TO BE USED IN DO LOOPS AND ARE OF THE FORMAT      */
  /*       DCL (STATING-VALUE, TERMINATING-VALUE, INCRE-   */
  /*        MENT VALUE                                      */
  /*    THE NUMBERS THEY REPRESENT CAN RANGE FROM           */
  /*      +/-.00000001 TO +/-99999999                       */
  /*    THE DATA CARDS SHOULD BE ARRANGED IN THE SAME       */
  /* ORDER AS THE GET LIST STATEMENTS FURTHER ON IN THE     */
  /* PROGRAM                                                */
  /*    EVERY TIME A GET LIST IS EXECUTED A NEW CARD WILL*/
  /* BE READ, THUS THE FIRST DATA CARD SHOULD CONTAIN THE*/
  /* VALUES OF 3, 3-LIMIT, 3-INC. EACH VALUE SHOULD BE   */
  /* SEPARATED BY ONE BLANK.  EACH VALUE SHOULD BE INSIDE*/
  /* THE RANGE OF NUMBERS STATED ABOVE.  THE SIGN IS ONLY*/
  /* NEEDED IF IT IS NEGATIVE.  THUS AN UNSIGNED NUMBER  */
  /* IS ASSUMED POSITIVE.                                 */
  /*    EXAMPLE:                                          */
  /*    COL1                                              */
  /*    L                                                 */
  /*    -.25 100 .25                                      */
  /*                                                      */
  /* THUS B = -.25, B-LIMIT = 100, B-INC = .25            */
  /* FOR FURTHER INFORMATION CONSULT A BOOK ON PL/1.      */
  /*------------------------------------------------------*/

1PROG:   PROC OPTIONS (MAIN):
         OPEN FILE (0) PRINT OUTPUT STREAM LINESIZE (132);

         DCL (B,B_LIMIT,B_INC) FLOAT(10):

         DCL (K,K_LIMIT,K_INC) FLOAT(10);
```

APPENDIX 1, CONTINUED

```
DCL  (A,A_LIMIT,A_INC)  FLOAT (10);

DCL  (L,L_LIMIT,L_INC)  FLOAT (10);

DCL  (C,C_LIMIT,C_INC)  FLOAT (10);

DCL  (XN,XN_LIMIT,XN_INC)  FLOAT 910);

DCL  (LOOP_CT) FIXED DECIMAL (5,0);

DCL  (A1,B1,PI,Q,N,ERROR)  FLOAT (10)

DCL(N_TEMP,A_TEMP,C_TEMP,L_TEMP,XN_TEMP) FLOAT(10);

1
  /* THIS SECTION READS IN ALL THE INPUT DATA */

  GET LIST (B,B LIMIT,B INC):
  GET LIST (L,L_LIMIT,L_INC);
  GET LIST (C,C_LIMIT,C_INC);
  GET LIST (XN,XN_LIMIT,XN_INC);
  GET LIST(N);
  GET LIST(ERROR);
*-------------------------------------------------------*/
*    THIS SECTION OUTPUTS THE LIST OF INITIAL DATA      */
*    +/-X.XXXXXXXE+/-XX                                 */
*    THERE IS 8 DECIMAL PLACE ACCURACY IN THE OUTPUT    */
*    FIELDS.                                            */
*-------------------------------------------------------*/

PUT FILE(0)  EDIT ('LIST OF INITIAL INPUT DATA')
                   (PAGE,COLUMN(52),A);

PUT FILE(0)  EDIT ('-------------------------------)
                  (SKIP,A)
                  ('-----------------------------------')
                  (A)  ('-----------------------') (A);

PUT FILE(0)  EDIT ('B = ',B,'B_LIMIT = ',B_LIMIT,'B_INC =
 ',B_INC)
          (SKIP(2),X(8),A,E(14,7),X(24),A,E(14,7),X(24),
           A,E(14,7));

PUT FILE(0)  EDIT('K = ',K,'K_LIMIT = ',K_LIMIT,'K_INC =
 ',K_INC)
          (SKIP(2),X(8),A,E(14,7),X(24),A,E,(14,7),X(24),
           A,E(14,7));
```

```
PUT FILE(O)  EDIT('A = ',A,'A_LIMIT = ',A_LIMIT,'A_INC =
 ',A_INC)
          (SKIP(2),X(8),A,E(14,7),X(24),A,E(14,7),X(24),
           A,E(14,7));

PUT FILE(O)  EDIT('L = ',L,'L_LIMIT = ',L_LIMIT,'L_INC =
 ',L_INC)
          (SKIP(2),X(8),A,E(14,7),X(24),A,E(14,7),X(24),
           A,E(14,7));

PUT FILE(O)  EDIT('C = ',C,'C_LIMIT = ',C_LIMIT,'C_INC =
 ',C_INC)
          (SKIP(2),X(8),A,E(14,7),X(24),A,E(14,7),X(24),
           A,E(14,7));

PUT FILE(O)  EDIT('XN = ',XN,'XN_LIMIT = ',XN_LIMIT,
 'XN_INC = ',XN-INC)
          (SKIP(2),X(8),A,E(14,7),X(24),A,E,(14,7),X(24),
           A,E(14,7));

PUT FILE(O)  EDIT('N = ',N)  (SKIP(2),X(8),A,E(14,7));

PUT FILE(O)  EDIT('ERROR = ',ERROR)  (SKIP(2),X(8),A,E(14,
 7) );
/*------------------------------------------------------*/
1
/*------------------------------------------------------*/
  N_TEMP=N;
  K_TEMP=K;
  A_TEMP=A;
  L_TEMP=L;
  C_TEMP=C;
  XN_TEMP=XN;

  DO B=B TOB_LIMIT BY B_INC;

     DO K=K_TEMP TO K_LIMIT BY K_INC;

         DO A=A_TEMP TO A_LIMIT BY A_INC;

            DO L=L_TEMP TO L_LIMIT BY L_INC;

               DO C=C_TEMP TO C_LIMIT BY C_INC;

                  DO XN=XN_TEMP TO XN_LIMIT BY XN_INC;

                     N=N_TEMP;

                     CALL CALUC;
```

```
                    END;
                 END;
              END;
           END;
        END;
 /*------------------------------------------------------*/
1
 /*------------------------------------------------------*/
 CALUC:  PROCEDURE;

       DCL P(N)   FLOAT(10);

       DCL V(N)   FLOAT(10);

       DCL X(N)   FLOAT(10);

       P(N)=(A-XN+C*D-B*K)/2*D;

       PI   = (-(D*P(N)**2)+(A*P(N))-(XN*P(N))+(C*D*P(N))-
                (A*C)+(XN*C));

       V(N)= PI   -((B*K*(P(N)-L))+(XN*B));

       X(N)=XN;

       D=1.24;

       B1=(-A+C+(B*K)-(2*B)/2;


       Q= (A-D*P(N)-X(N));

       IF P(N)<0 THEN P(N)=0;

       IF X(N)<0 THEN X(N)=0;

       IF Q<0  THEN Q=0;

       CALL HEADER;

       PUT FILE(O) EDIT (V(N),P(N),X(N),A,B)
           (SKIP,X(1),4(F(11,2),X(3)),X(2),F(4,2))
           (C,K,L,Q,PI)
           (X(5),F(11,2),X(6),F(4,2),X(4),2(F(11,2),X(3)),
            F(11,2));
```

```
        IF N=1 THEN RETURN;

1
 LOOP:   N=N-1;
         LOOP_CT= LOOP_CT +1;

         P(N)=(-2*D*A-K*X(N+1)+A*K-(D*C*K)+(2*D*X(N+1))+
          (L*K**2));
         P(N)=(P(N)-2*C*D**2)/(K**2-4*D**2));

         X(N)=((-K*(P(N)-L))+ X(N+1));

         V(N)=((-D*P(N)**2)+(A*P(N))-(X(N)+(D*C*P(N))+
          (X(N)*C));
         V(N)=V(N)-(A*C);
         V(N)= V(N)+V(N+1);

         PI = (-(D*P(N)**2)+(A*P(N))-(XN*P(N))+(C*D*P(N))-
          (A*C)+(XN*C));

         IF P(N)<0 THEN P(N)=0;

         IF X(N)<0 THEN X(N)=0;

         IF Q<0  THEN Q=o;

         IF LOOP_CT>59 THEN  CALL HEADER;

         PUT FILE (0)  EDIT(V(N),P(N),X(N),A,B)
            (SKIP,X(1),4(F(11,2),X(3)),X(2),F(4,2))
            (C,K,L,Q,PI)
            (X(5),F(11,2),X(6),F(4,2),X(4),2(F(11,2),X(3)),
             F(11,2));

         IF ABS(P(N+1)-P(N))>= ERROR & N>1 THEN GO TO LOOP;

 END CALUC;
 /*-------------------------------------------------------*/
1
 /*-------------------------------------------------------*/
 HEADER:   PROC;

 PUT FILE(0)  EDIT('VALUE','PRICE','RIVALS','DEMAND',
  'BEQUEST','COST')

         (PAGE,X(7),A,X(9),A,X(8),A,X(6),A,X(6),A,X(10),A)

         ('RESPONSE','LIMIT','CWN','PROFIT')

         (X(3),A,X(9),A,X(8),A,X(11),A);
```

```
 PUT FILE(O)  EDIT('OUTPUT','INTERCEPT','COEFFICIENT',
  'COEFFICIENT')

        (SKIP,X(34),A,X(5),A,X(2),A,X(14),A)

        ('PRICE','QUANTITY')

        (X(7),A,X(6),A);

 PUT FILE(O)  EDIT('-------------------------------------')
        (SKIP,A)
        ('---------------------------------------------')
        (A)  ('------------------------------')  (A);

   PUT SKIP;

   LOOP_CT =0;

 END HEADER;
 /*-------------------------------------------------------*/
 END PROG;
*DATA
.9 1_0 _1
.9 1 1 .2
90 100 10
45 50 5
20 25 5
0 10 10
50
.01
//
```

LIST OF CITATIONS

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2. Rifkin, E. and Sengupta, J. "Dynamic Limit Pricing under Stochastic Demand with Market Growth." University of California, Santa Barbara Department of Economics, Economics Working Paper #130, April 1979.

3. Gaskins, D. Jr. "Dynamic Limit Pricing: Optimal Pricing under Threat of Entry" Journal of Economic Theory Vol 3 September, 1971, pp. 306-322.

4. Bell, D. and Jacobson, D. Singular Optimal Control Problems. San Francisco, Academic Press, 1975.

5. Thanks to Radford Davis, Curt Mosso, Liz Caldwell, and Jim Weyman for assistance with the programming in this chapter.

SIMULATION

Simulation is undertaken for a variety of reasons. In our case, there are three motivations for using simulation:

- First, the sensitivity of the model can be studied.
- Second, the appropriateness of the particular specification will be examined.
- Third, we may use simulation techniques to study the meaning and implications of different optimal and suboptimal trajectories.

As was explained in Chapter 4, the original model is singular. Breaking singularity by invoking the uncertainty of entry estimation (and substituting perceived entry) yielded characteristic roots of opposite signs. By contrast, Rifkin and Sengupta [1] found two negative roots in their special case involving scaling without discounting or risk aversion. With discounting one may obtain two positive or two negative characteristic roots. The former would be nullified by a high discount rate. For the infinite horizon model the transversality condition dampens the explosive effect of positive roots. In the finite horizon analysis, the transversality condition can be reformulated to test the strength of its effect.

This latter sensitivity analysis can be done for a number of reasons. Gaskins [2] (and also Brock [3]) failed to interpret the transversality condition. As above, a finite discrete transversality condition can be interpreted as a bequest value. By varying this value, simulation evaluates the importance of the bequest term. While this is begun by Rifkin and Sengupta (op. cit.), a simulation study can extend their work and draw more comprehensive implications from the transversality condition. Their reference states that this transversality condition vitiates a positive characteristic root. Thus their system can not explode. The bequest term modifies the finite system in a less drastic manner. As below, it may require a large value to effect structural changes.

Another method of sensitivity analysis is the construction of a constrained series. Thus a simulated (or an optimal) series may be required to be within α % of the actual series for each actual value. As α decreases, the constraint becomes more effective. However, a zero value for α merely means the simulated series must recapitulate the actual series. An alternative constraint is to require the new series to be in the α neighborhood of the lagged actual series. This indicates an adjustment model with exogenous influences and once again reflects the dynamic nature of the series. These constraints can be applied in conjunction with varying of the essential economic coefficients.

Key economic coefficients in this Gaskins-style model include the demand slope, the response coefficient, the boundary condition for rival production, and the inventory proxy. Rifkin and Sengupta (op.cit.) show clearly the essential importance of the response coefficient. The effect of changes in the demand slope is less immediate in their simulations as other parameters varied simultaneously. The inventory proxies include stock effects (see Chow, [4]0 as well as pure inventory terms. The latter are suggested by Spence [5] as an entry deterrent. Unfortunately, multicollinearity prevented the separation of effects of stock income, and inventory terms. Accordingly the proxies were employed to retain explanatory power of these effects while reducing their standard errors. The boundary condition provides an anchor for the model. In economic terms the dynamic programming algorithm specified the final level of rival production. The DYNAMO series used initial level as a boundary condition. These boundary conditions can be varied to analyze the difference rival size implies. A zero value here means monopoly status in the model. Implications of this can be checked in a comparative statics framework. This will be done in detail below. The previous chapter indicated the model was robust over the economically acceptable region. It is anticipated that marginal variation of economic parameters would not vitiate the model.

The actual values selected for the simulation commence from a benchmark construction. The response coefficient .001 is used initially. Limit price bounded below by $12,000 (or 75% of IBM's minimum price) and bounded above by $16,000 (100% of said price), is set equal to $13,000. The entry equation is closed by specifying initial price at $18,000. Next the demand curve parameters must be specified. Here the selection was model 1 from equation 2.4e above. This is rewritten in the form

$$P = 6.66\ R_1 - 6.66\ Q - 6.66\ X.$$

The boundary condition for X is 150; that for Q is selected as 1500. It turns out that the X value terminates at 504. The intercept term (6.66 R_1) begins at 40,000 and grows at the rate 1% per quarter. One last specification is the zero value for the bequest coefficient. This is postulated for ready comparison with other results referred to above. Table 1 shows the quantity, rival quantity, and net price for the model with these benchmark values. For ready comparison Table 2 gives the corresponding values from the optimal series created via dynamic programming.

Next suppose that the response coefficient in the entry[1] equation were increased. Recall the crucial nature of this parameter suggested above. This is confirmed in Table 3. Not only does rival production rise much faster, but also market price and the dominant firm's production fall. The demand intercept increases steadily. In some seventeen years rival stock exceeds that of the dominant firm. Simultaneously rival profits exceed the dominant firm's profits. The DYNAMO printout shows this numerically and graphically This turnaround from the previous series suggests that the smaller response coefficient is more realistic as real world data show continuing domination by the major firm. This is approached by Plot 5 of Rifkin and Sengupta. The Gaskins value was much larger and stemmed from a more limited model. Brock's coefficient (.0075 quarterly) lies between the benchmark value and this alternate value.

Another change involves a switch to the Model 2 demand equation. Otherwise the benchmark model parameters apply. Here the other proxy (R_2) is used. From equation 2-5e the estimated form used here is

$$P = 3.85Q-3.85X+4.08R_2+184992 \text{ or } P = A-bQ-bX-cR_2$$

Since R_2 has negative values, this can be thought of as the

---

[1]Entry here may be identified with its proxy, rival market penetration, used in earlier chapters.

TABLE 1

QS	XS	PSN	T
2068	504	12658	20
2066	491	12647	
2064	479	12636	
2063	466	12625	
2061	454	12614	
2060	441	12604	15
2058	428	12593	
2056	416	12583	
2055	403	12572	
2053	391	12562	
2052	378	12552	10
2050	365	12542	
2049	353	12532	
2047	340	12522	
2046	328	12512	
2044	315	12502	5
2043	303	12492	
2041	290	12483	
2040	278	12473	
2039	265	12464	1

Notes: QS - Quantity Simulated

XS - Rival Quantity (X) Simulated

PSN - Price Simulated Net (of Cost = 12,000)

K = .001, b = .15, d = 0, $X_n \cong 500$

TABLE 2 - D.P. Values

T	Q	X	P
20	2987	500	31561
-	2857	481	30897
-	2768	463	30309
-	2681	446	29734
-	2597	429	29179
15	2514	413	28630
-	2434	397	28100
-	2355	382	27578
-	2278	367	27072
-	2203	353	26572
10	2130	339	26089
-	2058	326	25613
-	1988	313	25153
-	1920	301	24700
-	1853	289	24260
5	1788	278	23827
-	1724	267	23407
-	1662	257	22994
-	1602	247	22593
-	1542	237	22203

Notes: Dynamic programming series, with same parameters used as in Table 1.

TABLE 3

T	Q	X	P
20	1059	2552	18840
-	1081	2494	18981
-	1104	2434	19127
-	1127	2373	19277
-	1152	2310	19433
15	1177	2246	19494
-	1203	2180	19760
-	1230	2112	19933
-	1258	2043	20111
-	1287	1972	20295
10	1317	1899	20485
-	1348	1824	20681
-	1380	1747	20885
-	1413	1668	21095
-	1447	1588	21312
5	1482	1504	21537
-	1518	1419	21769
-	1556	1331	22009
-	1595	1241	22258
-	1635	1149	22514

Table 3: K = .01

second form. Along with the reduction in demand slope, there is also a decrease in the initial value of the demand intercept. Otherwise, initial values are unaltered. The results of these changes are shown in Table 4. Notice that rival production X rises more slowly than in the benchmark run. The price series and the dominant quantity of production each rise a little faster than in the basic series. Once again dominant firm profit and stock exceed profit and stock for the rival, as in the basic run. Comparison as in Rifkin and Sengupta is difficult because of the multiple changes made. This will be presented more fully in the comparative statics section.

Next the boundary condition will be varied. The terminal value drops to 368. The effects of this are to increase dominant production at the expense of rival production. Price has gone up. This is shown in Table 5. Again dominant stock and profit exceeded rival stock and profit. Here only the one boundary condition is varied from the benchmark model.

The remaining parameter is the bequest coefficient. Table 6 is identical with Table 1 for the quantities. Since net price equals market price, less 12,000, the prices are not significantly different. This uses a utility function with greater weight on profits in the terminal period (1.0) than on rival production in the

TABLE 4

T	Q	X	P
20	2392	378	21351
-	2385	370	21327
-	2379	361	21303
-	2373	353	21279
-	2367	344	21256
15	2361	336	21232
-	2355	328	21208
-	2349	320	21185
-	2343	312	21161
-	2337	304	21138
10	2331	295	21114
-	2325	287	21091
-	2319	279	21068
-	2313	271	21045
-	2307	263	21021
5	2301	255	20998
-	2295	247	20975
-	2290	239	20952
-	2284	231	20929
-	2278	223	20906

Notes: Model 2 used.

TABLE 5

T	Q	X	P
20	2137	368	26111
-	2135	355	26101
-	2134	342	26092
-	2132	329	26083
-	2131	316	26073
15	2130	302	26064
-	2128	289	26055
-	2127	276	26046
-	2126	263	26037
-	2124	250	26028
10	2123	237	26020
-	2122	224	26011
-	2120	211	26003
-	2119	198	25994
-	2118	185	25986
5	2117	172	25978
-	2116	159	25970
-	2114	146	25962
-	2113	133	25954
-	2112	120	25946

Note: Boundary condition changed.

TABLE 6

T	Q	X	P
20	2068	504	25658
-	2066	491	25647
-	2064	479	25636
-	2063	466	25625
-	2061	454	25614
15	2060	441	25604
-	2058	428	25593
-	2056	416	25583
-	2055	403	25572
-	2053	391	25562
10	2052	378	25552
-	2050	365	25542
-	2049	353	25532
-	2047	340	25522
-	2046	328	25512
5	2044	315	25502
-	2043	303	25492
-	2041	290	25483
-	2040	278	25473
-	2039	265	25464

Note: Bequest coefficient introduced, effect null. Compare Table 1.

following period (0.1). Changes in the weighting such as equal weights or greater weight to the bequest could alter the series.

As was mentioned earlier, simulation is widely used to observe the convergence to steady state values, if any. In the Gaskins model which has infinite time horizons, the steady state values were used for the following comparative statics result along the optimal trajectory

(i) $dP/d\gamma > 0$.

Further, Gaskins obtains the comparative dynamic result

(ii) $dP/dX_o < 0$.

In our case the validity of these results can be checked for different levels of finite horizons, different sets of parametric coefficients, and so on. To verify the Gaskins results above we shall translate them to our notation, select points from optimal paths, and demonstrate the suggested effects.

His growth term γ is equivalent to our autonomous income term A. The results to be confirmed are thus

(i) $dP/dA > 0$

(ii) $dP/dX_0 < 0$.

In Table 1 the first result is rapidly confirmed. As A increases, T increases. Let T increase from 1 to 2, then PSN increases from 12464 to 12473. This result for price (here net simulated price) can similarly be verified in other tables. Next a dynamic result of Gaskins can be confirmed. For the change in boundary condition X_0 examine Table 5. At T = 1, X_0 is 120 in Table 5, 265 in Table 1. The price from Table 5 to Table 1 decreases from 25946 to 24464, thus confirming the negative effect of increasing X_0. (The price term PSN was rendered comparable to the price in Table 5). Thus these Gaskins results can be confirmed in the simulation tables.

Table 7 compares the actual, dynamic programming optimal and DYNAMO stochastic simulation series. Means, standard deviations, variances and coefficients of variation are given for each series for varying horizons T (5, 10, 15, and 20 periods). Except for rival production, optimal means are greater than simulated means which are greater than actual means for the respective series. For standard deviations, the pattern is simulated series σ < optimal series σ< actual series σ (standard deviation), except for the very short run (T=5). The same pattern holds for the coefficients of variation. Thus both the dynamic programming optimal and the simulated optimal series are smoother than the actual series for prices, production, and profits. This smoothness result confirms Brock's findings.

In Table 8, Brock's series and ours are compared for relative efficiency in predicting market share. The respective actual series differ due to inclusion (Brock's) or non-inclusion (ours) of the smaller firms such as SDS, GE and RCA. The mean square error for Brock's market share forecast exceeds that for either quarterly model (Using either the dynamic programming series or the simulated series for optimal market share construction). Thus simulation data has here been used successfully in making accurate forecasts.

TABLE 7

P	QDP	QA	QS
5	18,323 (5)	1,171,713 (54)	7 (N)
10	61,838 (10)	1,216,947 (49)	24 (N)
15	121,239 (15)	1,221,914 (60)	49 (N)
20	190,234 (20)	1,170.205 (70)	82 (N)
	PDPN	PAN	PSN
5	1,691,782 (7)	30,299 (4)	302 (M)
10	3,456,113 (12)	25,299,850 (63)	1043 (M)
15	5,992,862 (17)	17,639,550 (56)	2168 (M)
20	8,971,418 (22)	14,837,040 (52)	3651 (M)
	XDP	XA	XS
5	784 (6)	6,376 (16)	391 (4)
10	2439 (12)	54,671 (39)	1,450 (9)
15	4533 (17)	77,152 (44)	3,170 (14)
20	6701 (23)	81,681 (48)	5,532 (19)

T	PIDPQ	PIAQ	PIDSQ
5	3.56E+13 (12)	2.46E+13 (56)	4.89E+09 (N)
10	7.80E+13 (21)	3.42E+14 (96)	1.65E+10 (N)
15	1.23E+14 (31)	2.64E+14 (109)	3.39E+10 (1)
20	1.64E+14 (41)	2.16E+14 (118)	5.67E+10 (1)
	PIDPX	PIAX	PIDSX
5	1.21E+12 (13)	1.77E+1 (20)	6.66E+10 (4)
10	2.56E+12 (23)	3.20E+13 (102)	2.45E+11 (9)
15	3.93E+12 (34)	2.37E+13 (91)	5.30E+11 (14)
20	5.04E+12 (45)	1.91E+13 (91)	9.17E+11 (20)

Notes: QDP - dominant firm production, optimal d.p. series
QA - dominant firm production, actual series
QS - dominant firm production, simulation series
PDPN- net price, d.p. series
PAN - net price, actual series
PSN - net price, simulation series
XDP - rival production, d.p. series

TABLE 7, Continued

Notes: XA - rival production, actual series
XS - rival production, simulation series

1.07E+07 = 1.07×10^7

N = Negligible

TABLE 7 Second Continuation (T=5)

Variable	Standard Deviation	Mean	Coefficient of Variation
QDP	135	2768	5
QA	1082	2018	54
QS	3	2064	N
XDP	28	464	6
XA	80	489	16
XS	20	479	4
PDPN	1301	17536	7
PAN	174	4332	4
PSN	17	12636	N
PIDPQ	5,968,386	48,674,883	12
PIAQ	4,963,768	8,864,080	56
PIDSQ	69,958	26,085,796	N
PIDPX	1,097,747	8,161,436	13
PIAX	421,765	2,128,752	20
PIDSX	258,095	6,050,392	4

TABLE 7 Third Continuation (T=10)

QDP	249	2562	10
QA	1103	2262	49
QS	5	2060	N
XDP	49	423	12
XA	234	604	39
XS	38	447	9
PDPN	1859	16,063	12
PAN	5030	7959	63
PSN	32	12,609	N
PIDPQ	8,834,212	41,570,796	21
PIAQ	18,505,641	19,367,244	96
PIDSQ	128,609	25,980,551	N
PIDPX	1,599,357	6,878,120	23
PIAX	5,660,046	5,557,181	102
PIDSX	494,577	5,641,291	9

TABLE 7 Fourth Continuation (T=15)

QDP	348	2372	15
QA	1105	1837	60
QS	7	2057	N
XDP	67	387	17
XA	278	633	44
XS	56	416	14
PDPN	2448	14,763	17
PAN	4200	7513	56

TABLE 7 Fourth Continuation (T=15), Continued

Variable	Standard Deviation	Mean	Coefficient of Variation
PSN	47	12584	N
PIDPQ	11,120,807	35,802,608	31
PIAQ	16,243,625	14,946,120	109
PIDSQ	184,070	25,878,898	1
PIDPX	1,981,223	5,860,663	34
PIAX	4,866,530	5,355,503	91
PIDSX	727,883	5,234,707	14

TABLE 7 Fifth Continuation (T=20)

QDP	436	2195	20
QA	1082	1555	70
QS	9	2053	N
XDP	82	354	23
XA	286	600	48
XS	74	384	19
PDPN	2995	13574	22
PAN	3852	7445	52
PSN	60	12558	N
PIDPQ	12,809,002	31,025,170	41
PIAQ	14,693,255	12,381,907	118
PIDSQ	238,038	25,779,776	1
PIDPX	2,244,839	5,040,878	45
PIAX	4,373,418	4,796,558	91
PIDSX	957,344	4,831,717	20

TABLE 8

Market Share Analysis

	BA	BP	E	A	DPO	SO	E1	E2
1963	.698	.730	-.032	.823	.865	.862	-.042	-.039
-	.683	.729	-.046	.792	.863	.844	-.072	-.052
-	.653	.725	-.072	.840	.859	.824	-.019	.016
-	.662	.720	-.058	.840	.857	.816	-.017	.024
-	.681	.714	-.033	.831	.855	.804	-.024	.027

Notes: BA - Brock actual series
BP - Brock's predicted series
E - Error + BA-BP (MSE = .0026)
A - Proposed actual series
DPO- D.P. Optimal series
SO - Simulated Optimal series
E1 - Error 1 = A-DPO (MSE = .0017)
E2 - Error 2 = A-SO (MSE = .0012)

As was mentioned at the outset, the third aspect of simulation deals with a comparative view of the optimal and suboptimal trajectories. In particular, we consider the following cases for comparison:

a) static short run myopic decision rule (MDR_1)

b) longer-run MDR_1

c) longer-run MDR_2

The myopic decision rule MDR_1 is computed by myopically maximizing profit one period at a time. Rival market penetration is ignored. A run is initiated where actual and simulated values coincide and proceeds forward in time.

By contrast the myopic decision rule MDR_2 recedes backward in time from the terminal period T. It is also built on myopic profit maximization, but is based on a two-period scheme. Every other period the dominant firm monitors rival output. Low inventory costs make such a policy feasible when marginal cost of inventory plus marginal cost of smoothed production together are less than marginal cost of unstable production.

The motivation for analyzing the alternative suboptimal paths is two-fold. One is that it relaxes the rigid assumption of the Gaskins formulation since a myopic rule does not pay as much attention to the entry equation as in Gaskins. In fact the formulation is very close to a monopoly firm model. The second motivation is that the time horizon which was so critical in the Gaskins model, which has to introduce the discount rate exogenously, is under-emphasized in the suboptimal trajectories.

Comparisons of the alternative myopic and non-myopic trajectories are made in terms of the following criteria, which are widely used in simulation of economic models (see Naylor et al. [6]). First, means are given so that optimal and myopic values can be readily compared. Second, standard deviations are given for comparisons of precision. Next, the coefficients of variation are reported. Loss functions are constructed to show the shortfall of myopic profits from optimal profits. Finally, mean square error is computed to quantify the divergence of myopic from actual profits, prices, and quantities for the dominant firm. Table 9 reports the comparisons for the short run MDR_1 analysis, Table 10 for a longer run. The backward rule MDR_2 comparison of optimal and myopic series is given in Table 11. Within tables it is seen that the mean for optimal variables exceeds that for myopic variables. However, the optimal output mean for the rival is exceeded by the actual mean. The optimal variables exhibit lower standard deviations than the myopic variables. Accordingly, the coefficients of variation are less for the optimal variables. Comparison across tables shows smaller losses in the short run. Further the one-period forward myopic series has smaller mean square error than the two-period backward myopic series. This contradicts the advantage postulated for the two-period backward series. The optimal series have the advantage of lower variances and greater means, which implies greater acceptance among risk averse firms. In this sense these series are optimal, and the resulting stability of profits at high levels emphasizes this point.

TABLE 9 MDR Short Run

X_A	X_O	Q_O	Q_m	P_O	P_m	πX_A	πX_O	πQ_O	πQ_m
386	386	2355	2355	27578	27578	6.01	6.01	36.69	36.69
778	397	2434	2287	28100	27211	11.83	6.39	39.19	34.79
703	413	2514	1786	28630	23881	8.35	6.87	41.81	21.22
1162	429	2597	2614	29179	29386	20.20	7.37	44.61	45.45
845	446	2681	2852	29734	30967	16.03	7.91	47.54	54.09
845	463	2768	2853	30309	30971	16.03	8.48	50.68	54.12
810	481	2857	1511	30897	22046	8.14	9.09	53.99	15.18
510	500	2937	2329	31561	27487	7.90	9.78	57.45	36.07

Loss function E Loss X = 323.75

E Loss Q = 2462.06

MSE(QM,QA) = 956,639

MSE(PM,PA) = 101,149,088

MSE(PIQA,PIQM) = $.834 \times 10^{15}$

TABLE 9 Continued

Variable	Standard Deviation	Mean	Coefficient of Variation
XA	234	755	31
XO	40	439	9
QO	205	2643	8
QM	477	2323	21
PO	1385	29498	5
PM	3174	27441	12
PIXA	3,328,916	4,034,519	66
PIXO	1,323,078	7,737,397	17
PIQO	7,269,458	46,494,853	16
PIQM	14,078,805	37,200,887	39

TABLE 10 Optimal vs. Myopic 1 period forward

X_A	X_0	Q_0	Q_m	P_0	P_m	πX_A	πX_0	πQ_0	πQ_m
290	290	1853	1853	24260	24260	3.56	3.56	22.72	22.72
1100	301	1920	1796	24700	23941	13.14	3.82	24.38	21.45
707	313	1988	607	25153	16036	2.85	4.12	26.15	2.45
1022	326	2058	1265	25613	20411	8.60	4.44	28.02	10.64
338	339	2130	2169	26089	14425	0.82	4.78	30.01	5.26
386	353	2203	2088	26572	25882	5.36	5.14	32.10	28.99
778	367	2278	2287	27072	27211	11.83	5.53	34.33	34.79
703	382	2355	1787	27578	23884	8.35	5.95	36.69	21.24
1162	397	2434	2614	28100	29386	20.20	6.39	39.19	45.45
422	413	2514	3965	28630	26367	6.06	6.87	41.81	56.97
422	429	2597	3965	29179	26367	6.06	7.37	44.61	56.97
422	446	2681	3965	29734	26367	6.06	7.91	47.54	56.97
422	463	2768	3965	30309	26367	6.06	8.48	50.68	56.97
810	481	2857	1511	30897	22046	8.14	9.09	53.99	15.18
510	500	2937	2329	31561	27487	7.90	9.78	57.45	36.07

E Loss X = 373.77

E Loss Q = 4247.06

$MSE_{QM,QA}$ = 1,040,392

$MSE_{PM,PA}$ = 70,069,031

$MSE_{PIQA,PIQM} = .628 \times 10^{15}$

TABLE 10 Continued

Variable	Standard Deviation	Mean	Coefficient of Variation
XA	280	732	38
XO	60	401	15
QO	310	2446	13
QM	479	2140	22
PO	2067	28191	7
PM	4566	25301	18
PIXA	3,249,549	5,349,809	61
PIXO	1,826,459	6,608,109	28
PIQO	10,151,607	40,197,557	25
PIQM	15,437,408	29,844,514	52

TABLE 11 Optimal vs. Myopic 2 period backward

XAM	XO	QO	QM	PO	PM	πXAM	πXO	πQO	πQM
500	500	2937	2937	31561	31561	9.78	9.78	47.45	57.45
500	481	2857	2937	31895	14601	1.30	9.09	53.99	7.64
422	463	2768	3065	30309	32379	8.60	8.48	50.68	62.46
422	446	2681	3065	29734	32379	8.60	7.91	47.54	62.46
422	429	2597	3065	29179	32379	8.60	7.37	44.61	62.46
422	413	2514	3065	28630	32379	8.60	6.87	41.81	62.46
703	397	2434	1787	28199	23884	8.35	6.39	39.19	21.24
703	382	2355	1787	27578	23884	8.35	5.95	36.69	21.24
386	367	2278	2087	27072	25882	5.36	5.53	34.33	28.97
386	353	2203	2087	26572	25882	5.36	5.14	32.19	28.97
1022	339	2130	1265	26089	20411	8.60	4.78	30.01	10.64
1022	326	2058	1265	25613	20411	8.60	4.44	28.02	10.64
1100	313	1988	1796	25153	23941	13.14	4.12	26.15	21.45
1100	301	1920	1796	24700	23941	13.14	3.82	24.38	21.45

E L X = 275.46
E L Q = 4562.10

$MSE_{QM,QA}$ = 1,358,708

$MSE_{PM,PA}$ = 102,834,077

$MSE_{PIQA,PIQM}$ = .119 x 10^{16}

TABLE 11, Continued

Variable	Standard Deviation	Mean	Coefficient of Variation
XAM	268	616	44
XO	60	401	15
QO	310	2446	13
QM	716	2324	30
PO	2067	28191	7
PM	5766	26152	22
PIXAM	3,677,383	5,027,795	73
PIXO	1,826,459	6,608,109	28
PIQO	10,151,608	40,197,557	25
PIQM	22,258,794	35,236,853	64

APPENDIX

The reader will recall references to other simulation packages. These were a FORTRAN program and a third-level SIMSCRIPT II.5 program. In some respects they are superior to the DYNAMO package, in others they are at a disadvantage. DYNAMO presented obvious facilities for handling dynamic problems. Attempts to move the demand intercept were less difficult in the DYNAMO program. Yet it lacks the stochastic entry and asset structure present in the other programs. Also, the others adapt more easily to a bequest term. DYNAMO does add agraphical display to its numerical output. Briefly, there are advantages and disadvantages for any of these simulation routines. For comparison, parametric values of the benchmark model are used in the runs shown here. In Table 12 (FORTRAN), notice that as rival production rises, dominant firm's production falls and price falls. The actual series showed dominant firm production increasing; however, the FORTRAN series lacks the proper growth term. This is also true of the SIMSCRIPT series. As with DYNAMO, comparison routines were run. It would be premature to list results, though, when the growth term is missing.

TABLE 12

T	Q	X	P
20	1837	506	24307
-	1843	493	24350
-	1830	520	24260
-	1850	479	24396
-	1864	452	24487
15	1855	469	24429
-	1844	490	24357
-	1871	437	24536
-	1884	411	24624
-	1892	395	24676
10	1919	341	24859
-	1937	305	24979
-	1955	270	25095
-	1955	269	25098
-	1960	259	25133
5	1974	232	25223
-	1985	208	25302
-	2002	174	25417
-	2009	160	25462
-	2015	150	24498

FORTRAN series, benchmark values.

TABLE 13

T	Q	X	P
20	1722	761	23466
-	1738	727	23578
-	1750	704	23657
-	1748	689	23706
-	1793	618	23944
15	1826	552	24160
-	1854	497	24345
-	1880	440	24522
-	1881	441	24530
-	1892	421	24598
10	1893	419	24606
-	1901	402	24662
-	1925	354	24823
-	1943	319	24939
-	1965	274	25087
5	1971	262	25126
-	1994	216	25279
-	2001	202	25329
-	2008	188	25373
-	2027	150	25500

SIMSCRIPT II.5 level 3 series.

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SUMMARY AND EXTENSIONS

This chapter provides some of the salient features of the applied model analyzed in Chapters 3, 4, and 5. It also describes some of the shortcomings of our analysis. It is my hope that some of the results developed in the earlier chapters and briefly summarized hereafter may be sharpened, improved, or generalized with more intensive work on relevant data sets and with the specification of alternative versions of the limit pricing model. Some of the alternative formulations which appear to me to be promising are also mentioned here with the hope that some future research work may be conducted either by myself or by others in the field.

Our analysis has been mainly concerned with developing an empirical formulation of the limit pricing model specified in Gaskins' theory [1] and related work. Gaskins' theory has been empirically analyzed by Brock [2] for the computer industry and in a simulation sense by Rifkin [3] to study the major predictive aspects of the model and its limitations. Our attempt extends this empirical formulation in three important directions. First we consider the econometric estimation of a discrete time version of the limit pricing model applied to the computer industry for the time period 1962-IV to 1968-IV, where questions of identification, economic meaningfulness of the respective signs of the coefficients and selection of the specification from a larger set of alternatives are considered. Second we have shown by alternative profiles of simulation the nature of optimal trajectory when some of the critical assumptions of the Gaskins model are relaxed; e.g., the implications of extending the horizon T, the impact of changing the response coefficient of the model and a contrast of the myopic versus non-myopic decision rules. Third, we have compared alternative versions of the optimal trajectory with the actual time path of state variables like quantity and profits and found that the optimal paths have more stability in the sense of coefficient of variation and mean square error and this feature is retained even when the period T is extended from 5 to 10, 15, or 20.

We have constructed data series and estimated econometrically equations for market penetration and demand. These have been tabulated in Table 4 and Table 5 of Chapter 3 respectively. The accepted values are the following:

RMP = .001 P

Q+X = -.15 P + 1.00 R1

and these values have been used in the simulations. The short run nature of the study is essential in this industry which has had a short history. The model may be viewed as a short run series of steady states. Without the correction for quality accomplished by the use of hedonic price indices, the model would have been much less successful. Better data have aided in the finding of results which are demonstrably superior to those of Brock (op.cit.). A calculation of mean square error for respective prediction of market share is reported in Chapter 5. This calculation shows the optimal path is more stable than the actual preferred by the dominant firm. Also the key nature of the response coefficient conjectured by Rifkin and Sengupta [4] can be confirmed by testing the values obtained in the rival market penetration studies reported in Chapter 3.

It is now appropriate to summarize our findings. The estimations of demand and rival market penetration were econometrically sound. The economic parameters thereby attained were succewsfully employed in a short run profit maximizing model a la Gaskins. This latter model retained economic viability when important parameters were varied. In several respects the model was shown superior to a myopic profit maximizing model.

Dynamic programming was used to check the shortcomings of linear specifications. The Gaskins model in our version is unstable for the infinite time horizon, unless a discount rate is introduced and the transversality condition applied. Therefore, dynamic programming algorithms were applied to analyze the stability of the optimal path with respect to certain crucial conditions or parameters of the problem such as the time horizon or the response coefficient. The simulated optimal trajectories we have computed show some of the stability aspects (see Chapter 5, Tables 1-7 and 12-13). Provided the empirical information were available, the dominant firm would, by implication, select the features of stability and smoothness of the optimal trajectory either due to risk aversion or aversion to fluctuating trajectories.

Third suboptimal myopic series were constructed for comparison with the optimal and the actual series. Tables 9-11 in Chapter 5 can be examined for the important features of the optimal-myopic comparison. The coefficient of variation is larger for the myopic series for all variables. Also the variance of optimal profit is less than that of myopic profit or the actual profit. This is further reinforced by the larger mean of the optimal profit series.

It would be appropriate here to mention some short-comings of our empirical analysis. The first is connected with the data systems for the computer industry where the product is not homogenous and technology is changing rapidly. Although we have used hedonic price indices and both single equation and simultaneous equation methods of estimation, we believe our results could be further improved if more reliable data with more appropriate disaggregation procedures are constructed. Second our basic specification closely follows the work of Gaskins' theory and related applications by Brock and Rifkin. There may be bias in the specification, not in the sense of statistical estimation, but in the initial choice of the models. To circumvent this aspect we attempted some alternative formualtions not reported in earlier chapters. We attempted to illustrate the leader-follower model but the results were not economically acceptable due to wrong signs obtained.

A few lines of possible extensions may now be mentioned here. The first approach by way of extension could consider the role of inventories and associated changes in capacity utiliation by following the theory of linear decision rules. This later theory has been successfully applied by Mills [5], Childs [6], and

Sengupta and Sfeir [7] for industries which are imperfectly competitive. The following specifications indicate the LDR hypothesis.

$$Prod = A_1 + B_1 Prod_{-1} + B_2 Inv_{-1} + B_3 UO_{-1} + B_4 O$$

$$Inv = A_2 + C_1 Prod_{-1} + C_2 Inv_{-1} + C_3 UO_{-1} + C_4 O$$

$$UO = A_3 + D_1 Prod_{-1} + D_2 Inv_{-1} + D_3 UO_{-1} + D_4 O$$

Here Prod is production, Inv inventory, UO unfilled orders, -1 indicates a first order lag, and O is a three period moving average of orders which may be taken to either lead or lag the other variables. However, in this formulation there are two major difficulties which must be resolved. One is in the concept of inventory itself which may partly be a state variable (unintended stocks, unfilled orders) or a control variable (holding of inventories for obtaining a smooth optimal path). The second is the assumption in Mills, Childs, and others that all the firms in the imperfectly competitive industry are all alike so that a supply curve may be meaningfully defined. However, that assumption may not hold, although the theory of LDR has been empirically applied to as heterogeneous industries as electical machinery (E.M.), primary metals (P.M.) and others.

The second line of extension is to specify other versions of the dominant firm behavior. For example, Stackelberg type leader-follower behavior, non-zero sum game theoretic models, and even analysis of conjectural variations. The concept of conjectural variations have been successfully estimated by Iwata [8] for Japanese industry data where some firms are dominant in the plate glass industry. It is clear that any development of this line of approach would have to incorporate a model for the rival also and include some of the following additional elements a--(a sharing of joint profits or a weighted combination of separate profits) b--(the threat point of non-cooperative point in the market sharing game) c--(other entry-preventing strategies besides price) and d--(elements of non-price competition which are partly reflected in conjectural variations). It is clear that realistic application of these alternative models would require data on costs and non-price strategies. (Incidentally, the study by Iwata of the plate glass industry in Japan utilized detailed cost data and data on other strategies to estimate oligopolistic interdependence).

A third line of extension may consider the stochastic aspects of the problem when perceived market penetration differs from the actual. We have mentioned before the discrepancy on this account which may partially resolve the singularity of the Gaskins' model. However, since perceived entry and penetration are both unobservable and anticipations data were not readily available for our purpose, we could not apply an econometric version of this model. This aspect has the implications for specifying a model of oligopoly behavior under conditions of risk and uncertainty which were analyzed by Rifkin and Sengupta (op. cit.) in their study. The later study may be empirically implemented with suitable proxies for risk aversion associated with fluctuating output or market shares.

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